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Field Measurements of the Roughness of Fault Surfaces

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ABSTRACT: We recorded the height of a granitic fault surface as a function of position along one-dimensional profiles. We show that the profiles exhibit an "anisotropic" scaling invariance: *self-affinity*. The difference between the maximum and the minimum height, and the standard deviation of the height, over a length L are proportional to L^ζ, where $\zeta \approx 0.84$. Other properties such as the Return Probability distribution or the Power Spectrum of the profile comfort this result. This self-affine property is in good agreement with recent works on artificial fractured surfaces. Previous studies at field scale are consistent with this concept.

INTRODUCTION

The morphology of fractured surfaces appears at first sight to be very different depending on the material, its fracture mechanism and the scale of observation. Nevertheless, it is fundamental to have a faithful description of the surface geometry — at least from a statistical point of view — in as much as it controls physical properties of the fracture such as

- *contact and friction* [Obuko and Dietrich, 1984] ; *e.g.* [Tullis, 1988] suggests a relationship between a distance parameter controlled by the surface roughness and a characteristic length of the friction law.
- permeability, since fracture aperture is directly related to the roughness of the two surfaces in contact [Brown, 1987; Brown, 1989; Thompson 1991].
- *wetting* [de Gennes, 1985]; the meniscus will be very sensitive to the detailed local morphology, and as a consequence, the flow of immiscible fluids in natural fractures, ...

Over the past ten years, a decisive progress has been acheived in the characterization of fracture morphology through systematic analysis of scale invariance properties, which were observed in a number of instances [Mandelbrot, 1982; Mandelbrot *et al.*, 1984; Termonia and Meakin, 1986]. The absence of a typical length scale is suggestive of some underlying critical phenomena [Charmet *et al.*, 1990; Hermann and Roux, 1990]. Brown and Sholz [1985], Power *et al.*[1987] showed the self-similar feature of natural fault surfaces. However, in spite of the versatility of fractal concepts, some care has to be taken when analyzing anisotropic structures (obviously directions along the mean fracture plane and perpendicular to it are not to be treated on the same footing). It has been recently reported that the geometry of brittle fractures was *self-affine* [Mandelbrot, 1985]. In such a case, concepts and tools originally developed for specifically self-similar object may fail [Mandelbrot, 1985; Wong, 1987; Brown, 1987; Brown, 1988].

The present letter reports on the analysis of statistical features of a natural fault surface at field scale using three different methods, and shows that indeed the surface is self- affine, and may be characterized by a roughness exponent very close to the one observed on fresh fracture surface of laboratory scale samples [Måløy *et al.*, 1992].

Let us recall some basic properties of self-affinity which are presented in more details in *e.g.* [Feder, 1988]. A profile $h(x)$ is self-affine if it is (statistically) invariant under the affinity

$$\begin{cases} x \rightarrow \lambda x \\ h \rightarrow \mu h \end{cases} \quad (1)$$

Group properties implies that μ should be an homogeneous function of λ. The homogeneity index ζ such that

$$\mu = \lambda^\zeta \quad (2)$$

is the *roughness* or Hurst exponent. Let us note that for a self-similar invariance, ζ is unity. In this case, both scaling factors are equal.

MEASUREMENTS AND SELF-AFFINE ANALYSIS

Using a field surface profiler, we have studied the topography of one natural rock surface. This instrument provides raw profiles (height function of position: $h(x)$), see figure 1a, of more than one thousand points with a sampling interval of 0.5mm. The x and h resolutions are respectively 0.1 and 0.2mm. The range of scale investigated is from 1 mm to 1 m. Eighteen independent profiles were recorded over a granitic fault plane at "Mayet de Montagne" (France)[Bernasconi, 1991]. They are transverse to the local direction of slip (at large scale, motion is not well defined). We report below the analysis of six of these profiles. Prior to any analysis, we substracted from the raw profile a linear function so as to reset the height of the first and last point to zero as shown in Fig. 1b. This defines a reference "mean" fracture plane. (Other choices have also been considered such as substracting a linear regression through the raw profile, and this does not affect significantly the results obtained).

Each profile is analyzed with three methods. The first one is a *"variable band width"* method [Feder, 1988]. A profile of length L is divided in windows or "bands" of width Δ indexed by the position of the first point x_0 of the band. The standard deviation of the height, w, and the difference δ between the maximum and minimum height on each band. Both w and δ are then averaged over the various band origins x_0 at fixed Δ. Band widths larger than $L/2$ are not considered because of insufficient independant sampling. Then $\langle w\rangle(\Delta)$ and $\langle \delta\rangle(\Delta)$ are plotted in a log-log diagram (see figure 2).

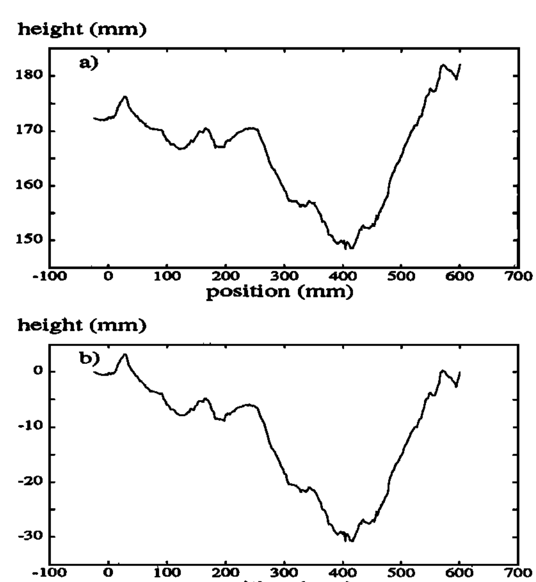

Fig. 1. (a) is one of the 6 raw profiles taken over the granitic fault surface. The height is plotted as a function of position. (b) is the filtered profile obtained by substracting the drift between the first point and the last one.

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This figure shows that both quantities follow a power law of Δ, as expected for self-affine where

$$\begin{cases} w \propto \Delta^{\zeta} \\ \delta \propto \Delta^{\zeta} \end{cases} \tag{3}$$

Therefore, the roughness increases continuously with the size of the window over which it is estimated. No absolute roughness scale can be defined independently of the sample size. An average of the estimates of ζ over the different profiles give $\zeta \approx 0.83$ and 0.9 respectively for w and δ.

The second method is the calculation of the return probability distribution. Starting from a point x_0 on the profile, we compute the distance d_0 where for the first time, the profile reaches the same height $h(x_0+d_0) = h(x_0)$ as that of the starting point. The distribution of d_0 for all starting is called the return probability distribution, $P(d_0)$. Figure 3 shows that this distribution follows a power law. For self-affine profiles, it can be shown [17] that the return probability histogram satisfies:

$$P(d_0) \propto d_0^{\zeta-2} \tag{4}$$

Our average estimate of ζ over all samples is approximately 0.83.

The third method is the calculation of the autocorrelation function of the profile through its Fourier transform. The power spectrum $P(f)$ of the profile is the Fourier transform of the correlation function $\langle h(x+\Delta x)\cdot h(x)\rangle$. Because of the filtering, a signal without any residual trend is computed with the FFT and so no artefact evolving like f^{-2} emerges in the power spectrum masking the physical information. Figure 4 shows that the power spectrum also follows a power-law. For self-affine profiles, we expect that the power spectrum fulfills [Feder, 1988]:

$$P(f) \propto f^{-1-2\zeta} \tag{5}$$

Table 1 summarizes the different estimates of ζ provided by the different methods of analysis and averaged over the 6 different samples. Typical fluctuations in the estimates are also indicated. Let us note however that these fluctuations are not to be confused with error bars on the determination of the roughness exponent. They do not take into account systematic deviations that could be due *e.g.* to finite size effects. Combining all different results gives a roughness exponent $\zeta \approx 0.84$.

In the variable band with method, we also considered other moments of the height distribution than the second. A nice power-law was also obtained, but all those moments could be described by a single ζ exponent, without having to resort to a more general multifractal description.

It is interesting to note that the *relative roughness* estimated over a sample size L, w/L decreases as the system size tends to infinity. Thus asymptotically, the fracture appears flat. This property justifies *a posteriori* the way we characterized the fracture surface through cuts, by probing the surface at constant intervals. Indeed since the relative roughness decreases to zero, overhangs if ever they exist at a very local level, would vanish upon coarse graining, in contrast with self-similar profiles, where the relative roughness is constant.

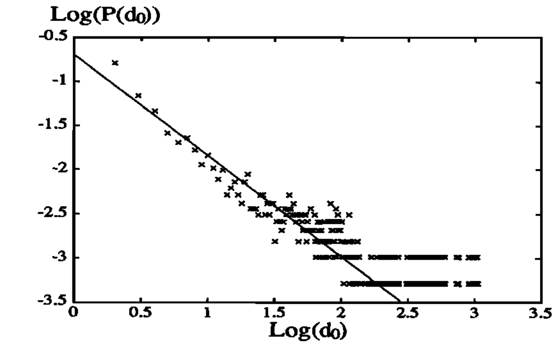

Fig. 3. The return probability histogram $P(d_0)$ in a log-log diagram for one profile. It is the distribution of intercept length of the profile with horizontal lines starting at each point of the profile. A best fit is shown as a straight line of slope -1.14 (giving $\zeta = 0.86$).

Discussion

Some previous roughness measurements were done by Brown and Scholz [1985] on laboratory and field scale samples. They recorded profiles on various surfaces: fresh natural joints, frictional wear surface formed by glaciation and a bedding plane surface at field scale. In this reference, the power spectra of the different profiles were studied and interpreted in terms of fractal dimension. The slope s of the power spectra provided a "fractal dimension" D defined by: $s = -(5-2D)$. It is straighforward to relate their result to the ζ exponent we introduced. Doing so we obtain a very consistent exponent of $\zeta \approx 0.8 \pm 0.1$ using Eq.(5), in very good agreement with our estimate.

The roughness of fractured surfaces expressed in terms of self-affinity was tested at laboratory scale on many different materials. Cracks surfaces on different rocks or minerals like sandstone, aragonite, magnetite or basic metamorphic rocks were analyzed and gave comparable results [Schmittbuhl, 1991]. Måløy *et al.*[1992] performed measurements over varied brittle materials including graphite, plaster of Paris, porcelain or bakelite and reported a roughness exponent of order 0.87. Also other fracture mechanisms were explored. For instance, in the ductile domain, the experiments on aluminium alloys of Bouchaud *et al.*[1990] provided again very similar observations: fractured surfaces appears as self- affine with an exponent of 0.8. It is quite striking that such a property appears to be so independent of the material type, the fracture mechanism, and the observation scale. The "universalily" of the self-affine nature of fracture surfaces seems, in our opinion, to be likely, or to be more conservative, we believe that the real precision on the roughness index that we may reasonnably defend, is such that most experimental data reported to date lies within the confidence limits of one single value – which depends on the space dimensionality. This "universality" implies that no information can deduced on the mode of fracture

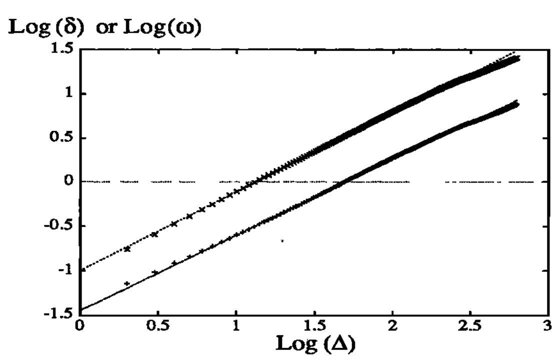

Fig. 2. The difference between the maximum and the minimum height δ (×) and the standard deviation $w = \sqrt{\langle h^2\rangle - \langle h\rangle^2}$ (+) on bands of width Δ are shown in a log-log diagram. The two straight lines are best fits of slopes 0.89 (··) and 0.85 (- -) respectively.

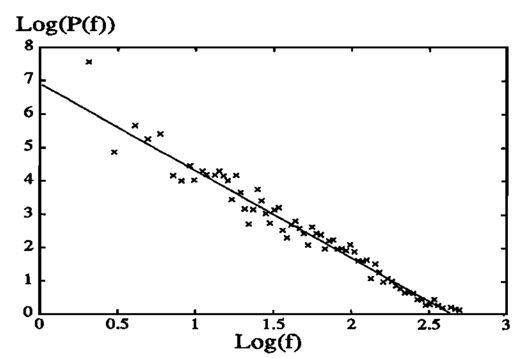

Fig. 4. The power spectrum $P(f)$ of the profile as a function of the freqency f in a log-log diagram. A linear regression of slope 2.60 is shown. The corresponding value of the roughness exponent is 0.80.

TABLE I. Estimates of roughness exponents and fluctuations

Measures	Average	Fluctuations
point number	1226	2%
δ	0.90	3%
w	0.83	2%
$P(d_0)$	0.83	2%
$P(f)$	0.81	3%

being given only the roughness index, but on the other hand, it also means that physical consequences derived from the self-affine geometry of cracks can be transposed to a wide class of different situations.

Albeit similar properties can also be obtained from numerical simulations in two dimensional systems [Hansen *et al.*, 1991], a theoretical understanding of such an "universal" scale invariance for *fractured surfaces* is still lacking. A qualitative analogy can be developed for perfectly plastic heterogeneous materials, where it can be shown — through a mapping onto an exactly solvable directed polymer problem in two dimensions [Kardar *et al.*, 1986; Kardar and Zhang, 1987] and onto a directed membrane problem in three dimensions — that a continuous interface of plastic elements develops with a self-affine geometry ($\zeta = 2/3$ in two dimensions).

In this paper, the field scale measurements of the roughness over a granitic fault were analyzed. One-dimensional profiles revealed with three methods, a scale invariance property of self-affinity characterized by an exponent ζ estimated to amount to 0.84. This result is consistent with previous studies on fault surfaces and some works on artificial fracture surfaces. Physical implications of this property are currently being investigated.

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