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A DROP DOES NOT FALL IN A STRAIGHT LINE: A RATIONALE FOR THE WIDTH OF STALAGMITES

J. Parmentier*[1], S. Lejeune[1], M. Maréchal[2], F. Bourges[3], D. Genty[4], V. Terrapon[1],
J.-C. Maréchal[2], and T. Gilet[1]
[1]*Department of Aerospace and Mechanics, University of Liège, Liège, Belgium*
[2]*BRGM, University of Montpellier, Montpellier, France*
[3]*Géologie-Environnement-Conseil, Saint-Girons, France*
[4]*EPOC, University of Bordeaux, Bordeaux, France*

Summary Drops loaded in calcium ions detach from stalactites and impact the underlying stalagmites, thereby allowing these latter to grow through calcite precipitation. Nevertheless, little is known about the influence of the drop free fall and splash dynamics on stalagmite shape and width. Through high-speed imaging of impacting drops on stalagmites from several caves, we observed that the impact point position of the drops is scattered, sometimes over several centimetres. We show that this dispersal has no external cause and must, therefore, be self-induced. Using a Langevin-like equation, to describe the free fall in response to gravity and aerodynamic forces, we then propose a prediction of the impact point dispersal as a function of the falling height travelled by the drops. We finally show that measured stalagmite widths are correlated to the impact point dispersal of the drop.

INTRODUCTION

Beyond their outstanding beauty, stalagmites help to understand past climate and hydrology. Measuring e.g. the size variations of the annual laminae seen in vertical cross sections of stalagmites give information on the precipitation history in regions where ice or sediment cores cannot be used [1]. While the chemical reactions underlying the slow growth of the stalagmites is fairly well understood, little is known about the fluid dynamics at play during their formation. The calcite is brought by drops detaching from the stalactite. The drops fall and impact a very thin film of water that covers the stalagmite. For most stalagmites, the drop velocity is sufficient to generate a splash, with the formation of a corona and secondary droplets (Figure 1).

In previous models of stalagmite growth, it was commonly accepted that drops fall on a straight vertical line from the stalactite, thereby feeding the stalagmite film from one central point [2]. However, our high-speed movies revealed that the impact points of drops originating from a single stalactite are scattered, sometimes over several centimeters (Figure 2).

MATERIALS AND METHODS

We have taken high-speed movies of drops splashing on a wide variety of stalagmites as well as still pictures of stalagmites only, in seven different caves of the south of France. The radius, velocity and impact point of the drop, along with the radius of the spreading lamella formed at impact and the stalagmite width, were measured by image processing. Additional data were also taken in a more controlled lab setting, for a drop radius similar to that encountered in natural conditions.

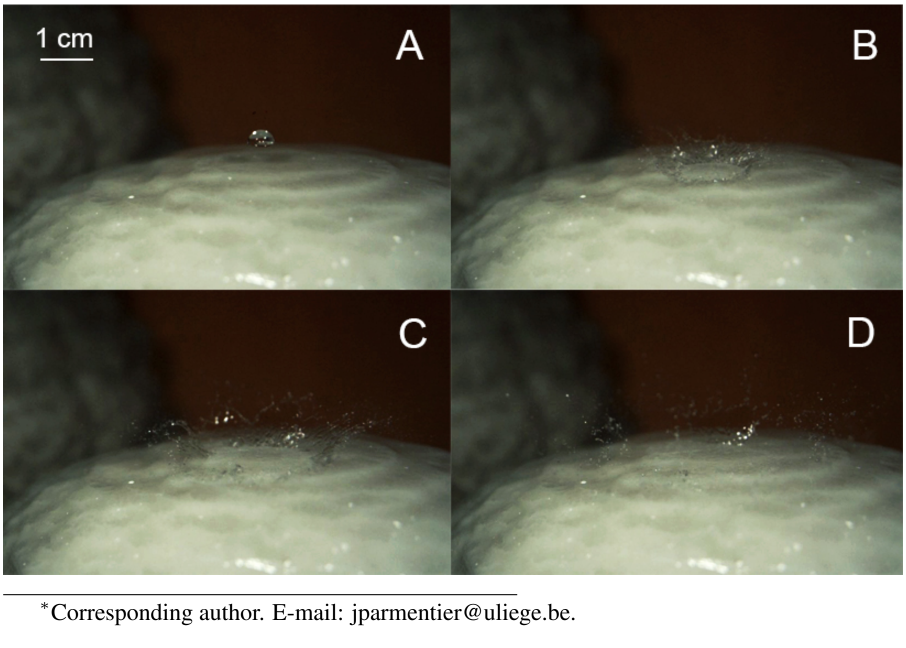

Figure 1: (A-D) Time sequence of the impact of a drop on a stalagmite (Orgnac cave, South of France), with lamella spreading and crown formation. All frames A-D are separated by 1.9 ms. (A) The drop of radius 2.61 mm at most 625 μs before the impact. (B) Formation of the crown at least 625 μs after the drop crushed on the stalagmite. (C) Spreading of the lamella in the water residual film and ejection of secondary droplets. (D) Fragmentation of the crown. The lamella has reached the stationary value $r = 2.3$ cm, around 7 ms after the initial impact.

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*Corresponding author. E-mail: jparmentier@uliege.be.

RESULTS

Our measurements show that drops falling from the ceiling of caves all have the same radius and that the size of the fluid lamella formed by their impact on the stalagmite is weakly dependent on the drop velocity. On the other hand, drops falling from higher ceilings are faster and more dispersed. We postulate that drops do not fall straight, but rather follow a wavy trajectory owing to their aerodynamic interaction with the surrounding air. This hypothesis does not come out of the blue; the same phenomenon is also observed in a controlled lab environment, where the falling drops are protected from parasitic air currents. Furthermore, the airflow measured in caves could only cause a deviation of a few millimeters at most.

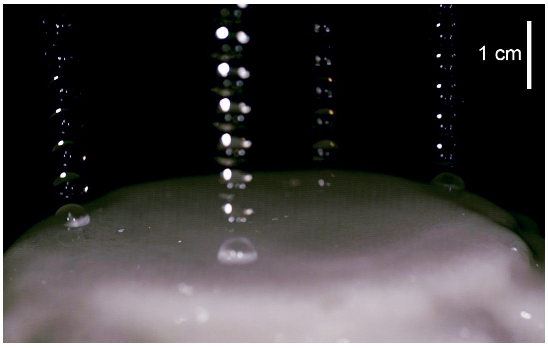

Figure 2: Drops coming from the same stalactite fall in different locations on the stalagmite below (Orgnac cave, South of France). This image was obtained by superimposing frames showing the trajectories followed by four drops landing on this stalagmite. For each drop the frames are separated by 625 μs, from the moment the drop appears in the field of view, up to right before impact.

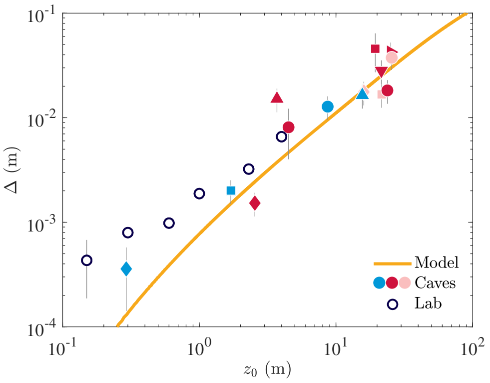

Figure 3: Standard deviation Δ of impact point position as a function of the falling height z_0. The solid curve is obtained by solving numerically the recurrence relations obtained for the drop position and velocity. Hollow symbols correspond to data from a lab setting, and solid symbols to in situ measurements, both obtained with high-speed videos. (Credit: The Royal Society 2019 [3])

The aerodynamic force acting on a drop considered spherical, can be formulated as follows:

$$\mathbf{F} = -C_d \frac{\rho_a S \left|\dot{\mathbf{X}}\right|}{2} \dot{\mathbf{X}} - C_l \frac{\rho_a S \left|\dot{\mathbf{X}}\right|}{2} \mathbf{e}_r \times \dot{\mathbf{X}} \tag{1}$$

where ρ_a is the air density, S is the cross-sectional area of the drop perpendicular to its motion, $\mathbf{e}_r$ is a unit vector of random direction in the plane perpendicular to the velocity vector of the drop, $\dot{\mathbf{X}}$, and C_D and C_L are the drag and lift coefficients, respectively. The first contribution of this force, the drag, is responsible for the saturation velocity reached by drops falling from very high ceilings ($z_0 > 20$ m). The second term of the equation represents a lift force which originates from the wake of the drop. Above a Reynolds value of around 212, which is still laminar though, the break of symmetry in the wake of the drop induces the shedding of randomly oriented vortices at a frequency increasing with the drop velocity. It is assumed that each of these vortices pushes the drop in a direction opposed to that of its emission, therefore deviating the drop from its original position and creating a random walk-like trajectory.

Integrating the horizontal projection of Newton's equation over one vortex shedding period yields two discrete equations for the position and velocity of the drop. An ensemble average over many falling successive drops allows to estimate the horizontal dispersal as a function of the falling height of the drop (comparison with lab and cave measurements in Figure 3).

The width of several dozens of stalagmites is finally related to the size reached by the liquid lamella formed at impact and to the impact point dispersal. For small falling heights, the stalagmite width is mostly set by the maximum spreading of the lamella, while for large falling heights, the stalagmite radius increases proportionally to the impact point dispersal.

CONCLUSION

We show that there is a strong positive correlation between the falling height, the impact point dispersal and the stalagmite width. We propose a theoretical model of the fall of the drop that includes aerodynamic forces. Owing to some lift of random direction that represents the effect of vortex shedding, the drop experiences a random walk, and its impact point on the stalagmite is scattered.

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