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Derivation of the Darcy-scale filtration equation for foam flow using the volume averaging technique

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1 Introduction

Physically, foam is represented as a two-phase system, where gas bubbles are dispersed in a liquid phase and are separated by thin borders, which are called lamellae. When the bubble-size is much smaller than the characteristic pore size, foam in porous media can be considered as a single non-Newtonian fluid phase, as is the case for bulk foam. The rheology of foam is complex and depends on several characteristics as bubble size, quality, stability, etc. To avoid all these complexities and simulate foam transport in porous media, the "population balance" approaches were suggested by several authors [3]. Generally, there are two main approaches of foam modelling: Local-Equilibrium models (empirical, semi-empirical) and Bubble Population-Balance models (mechanistic). In addition, there are some other modelling approaches, like percolation theory, fractional flow theory, catastrophic theory, and filtration theory. Both Local-equilibrium and Bubble Population-Balance models are designed by a modification of Darcy's law parameters such as gas relative permeability, as well as gas viscosity. Application of these approaches needs some general assumptions. Mathematically, the relative permeability and the viscosity of foam in porous media are grouped together and mentioned as gas phase mobility in Darcy law, but physically they are separable.

The main objective of this study is to derive a 3D Darcy-scale momentum equation for liquid foam (surfactant, water and air) flow using the volume averaging method, which allows modelling foam injection process at the local or column scale experiments. In particular, the impact of the scale on pressure gradient threshold above which the foam flows will be determined.

2 Upscaling of foam flow in porous media

The upscaling process is a very important procedure in which the static and dynamic characteristics of a pore-scale model should be taken into account in the local-scale model. Certainly, the direct modelling of fluid flow at the pore scale in a porous medium sample is practically impossible due to the complexity of the pore structure and the huge demand for computing resources. Therefore, it is necessary to derive a local scale model. Upscaling and modelling the flow of non-Newtonian fluids through porous media are involved in many problems substantial to science and industry. Appreciable examples are in biological systems, oil recovery operations, food processing, cosmetics, textile, paper industries and also in soil remediation technologies. For Newtonian fluids, the flow through porous media is modelled by Darcy's law, which has been theoretically derived from the Stokes equation via homogenization [1] and the volume averaging methods [4]. However, the filtration laws for non-Newtonian fluids are more complicated due to their complex rheology and the complex microstructure of porous media. In our study, we investigate the pore to Darcy scale upscaling of the flow of foam as a non-Newtonian, one phase fluid, using the volume averaging method. The apparent viscosity of bulk foam was first measured with a rotational rheometer (Haake Mars 60, ThermoFisher Scientific Company). As shown in Figure 1a, the apparent viscosity of foam fits well the Herschel-Bulkley model for $\tau \geq \tau_0$,

$$\tau = \tau_0 + k\dot{\gamma}^n \tag{1}$$

where τ [Pa] is the shear stress, τ_0 [Pa] is the yield stress, $\dot{\gamma}$ [s^{-1}] is the shear rate, k [Pa s^n] is the consistency index, and n is the flow index [2].

If τ_0 is zero, the model alters to the power law model, which is first investigated here. Application of the volume averaging method to the pore scale equations, without yield stress, leads to the following macroscopic equation:

$$\langle \mathbf{v}_\beta \rangle = -\frac{\mathbf{K}_{app}\cdot\left(\nabla\langle p_\beta\rangle^\beta - \rho_\beta \mathbf{g}\right)}{\left(k\left\|\langle \mathbf{v}_\beta\rangle\right\|^{n-1}\right)} \qquad (2)$$

Equation 2 is a generalized Darcy's law for the β-phase considered as a power law fluid, where $\langle \mathbf{v}_\beta \rangle$ is the superficial average velocity, $\left\|\langle \mathbf{v}_\beta\rangle\right\|$ is the magnitude of superficial average velocity, $\langle p_\beta\rangle^\beta$ is the intrinsic average pressure, ρ_β is the density, $\mathbf{g}$ is the gravity acceleration, and $\mathbf{K}_{app}$ is the apparent permeability which depends on the geometry and the direction of the flow velocity. The apparent permeability in longitudinal direction was obtained by solving the associated closure problem in a simple isotropic cylindrical geometry. The variation of the apparent permeability versus porosity for a Newtonian and a non-Newtonian fluids are presented in Figure 1b. We found that the apparent permeability of non-Newtonian fluid is also increased with increasing the porosity, as a Newtonian fluid. The values of apparent permeability for non-Newtonian fluid are about two orders of magnitude smaller than values for Newtonian fluid.

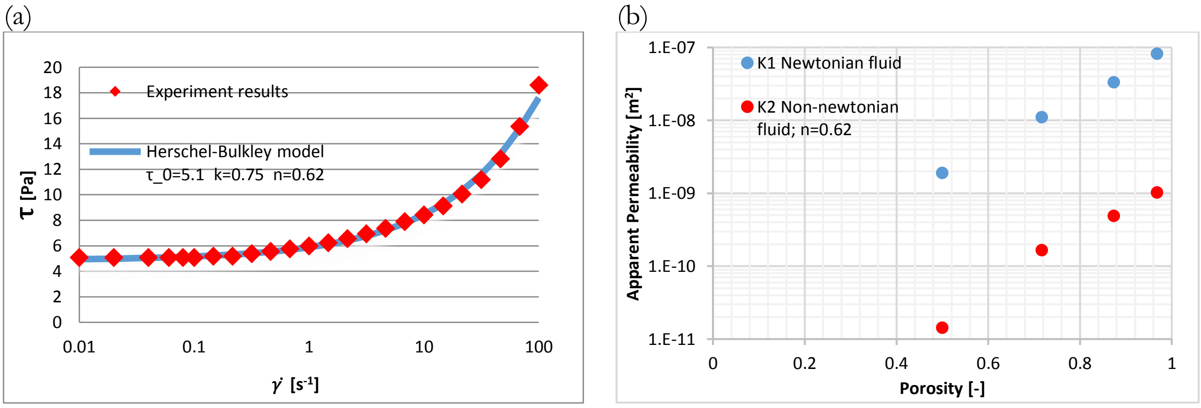

Figure 1: (a) Non-Newtonian foam behaviour (10*CMC solution of Triton X-100, Foam quality 85%, T=20 °C) and (b) Apparent permeability of Newtonian and non-Newtonian fluids.

3 Perspectives

Currently, we have almost developed a macroscopic model for power-law, non-Newtonian fluid flow in porous media. Besides, we are working in order to take into account the yield stress term in the up-scaled equations. In addition, validation of the macroscopic model by comparison with pore-scale, direct numerical simulation and also the experiments will be presented during the conference.

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