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## A Residual Distribution method for the Shallow Water equations in ALE framework on the sphere

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We consider the numerical approximation of the Shallow Water Equations (SWEs) in covariant curvilinear coordinates, in view of application to large scale hydrostatic wave phenomena, such as the propagation of tsunami waves. To provide enhanced resolution of the propagating fronts we consider adaptive discrete approximations on moving triangulations of the sphere. To this end, we re-state all Arbitrary Lagrangian Eulerian (ALE) transport formulas, as well as the volume transformation laws, in generalized curvilinear coordinates. Using these results, the SWEs can be written in a framework in which points move arbitrarily in a curvilinear reference frame. We then discuss the implementation of a multidimensional upwind scheme known as Residual Distribution (RD) in order to discretize the resulting ALE Shallow Water equations on the sphere. At the discrete level one must consider the preservation of time accuracy, non-linear stability but also the preservation of important physical steady states on moving meshes. A naive extension of fixed grid methods may lead to spoil the above properties and to the rise of numerical instabilities. For this reason classical properties as the Discrete Geometric Conservation Law and the C-property are reformulated in the more general context of moving curvilinear coordinates. The proposed RD method is tested on standard benchmarks for the SWEs on the sphere and it is compared to a classical Finite Volume method, both in the fixed grid case and in the ALE moving mesh case.

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