

# Bayesian Networks for the derivation of probabilistic functionality loss curves for bridge systems

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► **To cite this version:**

Pierre Gehl, D D 'Ayala. Bayesian Networks for the derivation of probabilistic functionality loss curves for bridge systems. 16th World Conference on Earthquake Engineering, Jan 2017, Santiago, Chile. 2017. <hal-01391952>

**HAL Id: hal-01391952**

**<https://hal-brgm.archives-ouvertes.fr/hal-01391952>**

Submitted on 4 Nov 2016

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## BAYESIAN NETWORKS FOR THE DERIVATION OF PROBABILISTIC FUNCTIONALITY LOSS CURVES FOR BRIDGE SYSTEMS

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### **Abstract**

In the context of infrastructure risk assessment, the application of fragility curves to elements such as bridges should mostly serve the purpose of quantifying the performance losses at the system level (e.g. disruption of traffic, additional travel times), since these losses usually outweigh the direct costs associated with the physical damage of infrastructure. To this end, a methodology is proposed for the derivation of probabilistic functionality curves for bridge systems: these curves directly provide the probability of exceedance of various loss metrics given the level of seismic intensity. The main steps of the proposed approach are the following:

- Identification of the failure modes for the various components of the bridge system (e.g. piers, bearings, deck, abutments, etc.).
- Derivation of specific component fragility curves for each component damage state.
- Estimation of the functionality losses that are associated with each component failure mode, through an expert-based survey.
- Construction of a Bayesian Network that describes the failure of the system, from the seismic intensity to the component damage states and the subsequent functionality losses.
- Use of the Bayesian Network to generate the joint probability of occurrence of various levels of functionality losses given the seismic intensity.

This approach is then applied to a generic multi-span simply-supported reinforced concrete bridge, for which component fragility curves are analytically derived through non-linear time-history analyses. The considered loss metrics are the repair duration, the proportion of closed lanes and the speed limit reduction, so that these parameters can be directly fed into traffic modelling tools for the computation of induced delays and the optimization of restoration strategies.

*Keywords: bridges, Bayesian Networks, functional losses, restoration, fragility curves*



## 1. Introduction

In the context of infrastructure risk assessment, the derivation of fragility curves for physical elements such as bridges constitutes a key step of the process, since it leads to the quantification of damage maps for a given earthquake scenario. Most of the fragility curves that are found in the literature [1] are based on global damage scales, which are specified by the successive damage states of the bridge's structural components. While such damage scales are suitable indicators of the severity of the damage, they are not necessary consistent in terms of induced losses (e.g. functionality losses, repair durations, etc.). However, in the case of network systems, it has been shown that assessing the system performance in terms of functionality loss (e.g. disruption of traffic, additional travel times, etc.) provides more accurate information than solely quantifying the direct costs associated with the physical damage of infrastructure [2].

To this end, some studies propose a mapping structure between the physical damage states and the functional losses, however the results are usually based on a single structural component type such as piers [3,4] or they are provided with insufficient justifications [5]. Therefore an original method is proposed in the present study, where the different bridge component failure modes are identified and directly associated with various levels of functionality losses. Based on their component fragility curves, the joint probability of occurrence of the different component failure modes can then be quantified through system reliability tools such as Bayesian Networks (BNs) [6]. It is then expected to derive probabilistic functionality curves by defining discrete ranges of functional losses (e.g. probability of exceeding a given repair duration or a given percentage of lane closure for a given seismic intensity level). Such curves will have the double merit of directly providing functional losses for a subsequent network analysis and avoiding the definition of a consistent damage scale, since all component damage states can be harmonized in terms of functional consequences.

Section 2 of the paper details the methodological framework, where the component failure modes are defined along with their corresponding functional consequences. The formulation of the BN that is required to assemble the component fragility curves is then summarized. Finally, the method is applied to an example bridge system in Section 3, where the abilities of the BN in terms of forward and backward analysis are demonstrated.

## 2. Methodological framework

The proposed approach is based on the following steps, which are then detailed in the subsections below:

- Identification of the bridge's structural components and the corresponding damage mechanisms;
- Association of each component failure mode with a set of functional losses, through an expert-based survey;
- Formulation of a BN structure in order to update the probabilistic distributions of losses at the system level.

### 2.1 Component failure modes

When reinforced concrete highway bridges are considered, the most common structural components are piers (single or multi-column), bearings (fixed or expansion devices) and abutments. It may be argued that deck spans should be included in this inventory, however it is considered here that these components remain in the linear elastic range, which is a common assumption due their high rigidity and the nature of their connections (i.e. especially in the case of multi-span simply-supported decks).

A literature review of qualitative damage scales has led to the inventory of the types of damage states or failure modes that are likely to affect each component (see Table 1), based on the studies by Cardone [7], Nielson [8] and Tsionis et al. [1]. Elastomeric bearings with steel dowels are considered, where damage can occur for different deformations levels. In the case of piers, flexural and shear behaviours are in competition; while abutments can experience passive or active behaviours, depending on whether the backfill soil or just the piles are solicited.



Table 1 – Inventory of possible failure modes for the bridge components considered

ID	Component	Failure mode	Damage ‘Severity’	Description
1	Pier	Bending	D1	Minor cracking/spalling and first yielding
			D2	Cracking/spalling (still structurally sound)
			D3	Column degrading without collapse (structurally unsafe)
			D4	Column collapse or reinforcement buckling
2	Pier	Shear	D3	Brittle shear failure
3	Abutment	Piles (active)	D1	Minor cracking/spalling
			D2	First yielding point
			D3	Ultimate deformation / Vertical offset
4	Abutment	Backfill (passive)	D1	Gap closure
			D2	Passive resistance of backfill soil is reached
			D4	Ultimate displacement of the backfill system
5	Elastomeric bearing	-	D1	Noticeable deformation
			D2	Possible deck realignment and dowel fracture
			D3	Girder retention and deck realignment
			D4	Deck unseating

The field describing the damage ‘severity’ represents the global damage state (i.e. at the level of the bridge system) that is usually considered to be reached when the given component is damaged through the mentioned failure mode. This classification directly results from the analysis of the qualitative damage scales that have been defined for bridges in the literature. Therefore one may question the consistency of this classification in terms of functional consequences, for instance due to the presence of component damage states that may induce disproportionate losses.

## 2.2 Functional consequences

It is proposed to represent the functional consequences as two distinct metrics, i.e. (i) the repair duration and (ii) the loss of functionality in terms of lane closure or speed restriction. The first measure is useful to conduct time-dependent scenarios with various restoration strategies and to estimate the duration during which the network system is likely to be disturbed. The second one is essential to assess the reduced capacity of the bridges, which can be used to run a traffic model on the degraded network system.

In order to efficiently quantify these loss metrics, it is necessary to consider the physical damage states at the component level, since it has been seen that each component type has very specific failure mechanisms. Therefore it is proposed to associate these loss metrics to each of the component failure modes summarized in Table 1. To this end, it is proposed to build these loss metrics by using expert judgement: this decision is motivated by the lack of empirical data of infrastructure failure events, especially in the European context. Pending further numerical or experimental investigation on the calibration of these loss metrics, the results of a basic expert-based survey are used, as detailed below, with the sole purpose of providing preliminary data for the demonstration of the potential of the proposed approach.

A survey form has been sent to infrastructure managers and experts within the INFRARISK consortium [9], with the objective to quantify the two loss metrics that correspond to each component failure mode. Three groups of experts have provided answers to the survey: they have around twenty years of engineering and construction practice, in various technical departments and geographical areas.

Unlike what is recommended in Cooke’s method [10], no seed questions have been submitted to the experts, thus preventing the use of weights in the answers. Unfortunately, this strong limitation is imposed by the modest amount of experts that have taken part in the study. An extended group of experts could not be mobilized due to the difficulty to find experts whose knowledge spans all failure modes and to the necessity to keep these developments within the project’s consortium.

The intervals of loss values proposed by the different groups of experts are then reconciled by building a probabilistic functionality model for each type of loss metric: an empirical cumulative distribution function can be assembled for each component damage state, as shown in Figure 1, using equal weights. This construction corresponds to the ‘pooling’ step that has been mentioned in Cooke’s method.

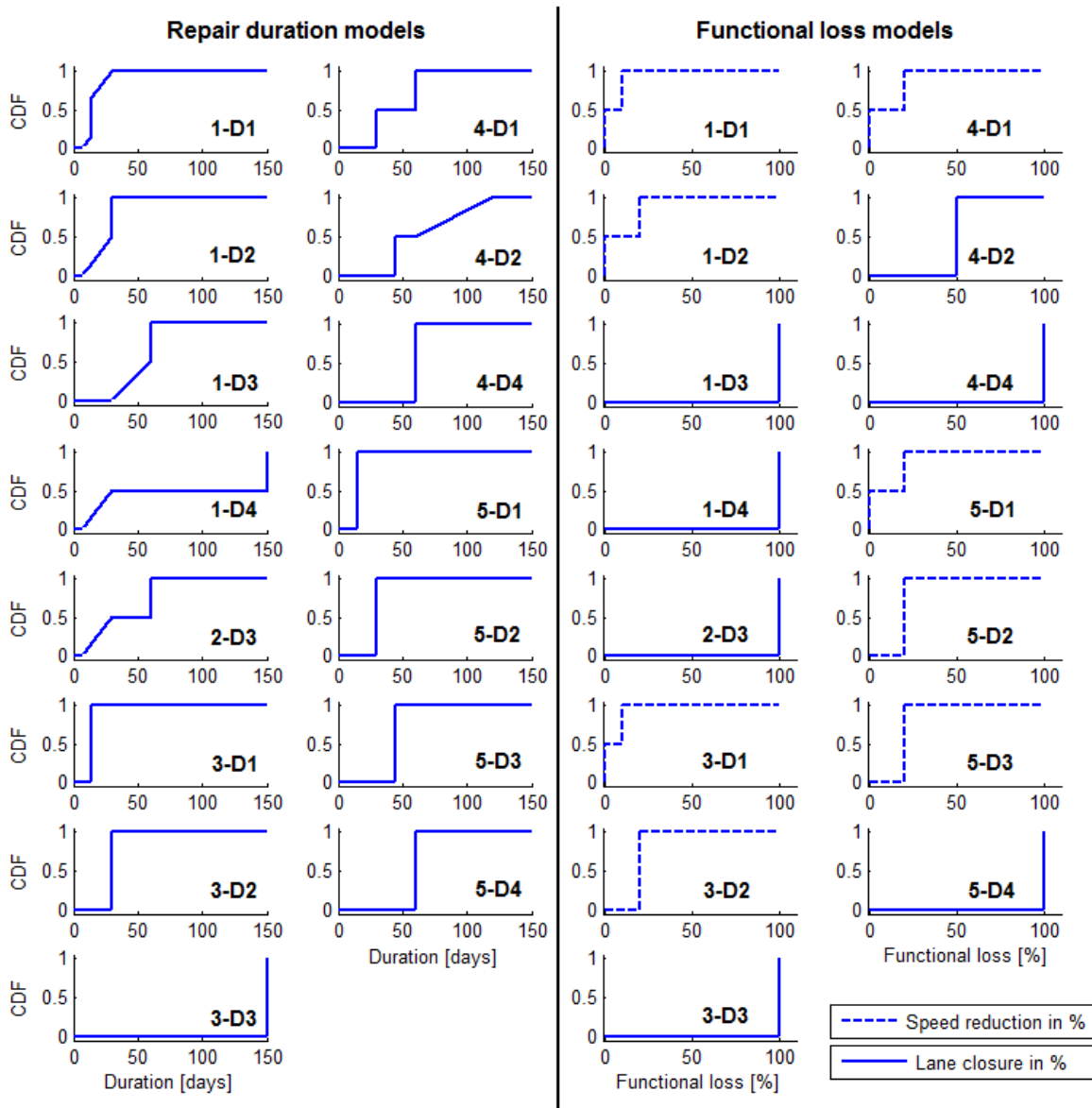


Fig. 1 – Expert-based repair duration (left) and functional loss (right) models for each component failure and damage state identified in Table 1

For heavier damage states the lane closure metric takes precedence, while the speed reduction metric may only be used when there is no lane closure (i.e. lighter damage states). The loss models are very coarse and it can be seen that most of them are only steps functions, since intervals of values have not been proposed by all



experts. Therefore, pending a refinement of the loss models, these values should only be considered for illustrative purposes, i.e. the demonstration of the proposed method.

### 2.3 Bayesian Network analysis

While matrix-based system reliability methods have been introduced by Song and Kang [11] and Kang et al. [12] for the fragility assessment of a system of interdependent components, Gehl and D'Ayala [6] have shown that a BN formulation can be run more efficiently with a better ability to treat large and complex systems. The statistical dependence between the component damage events is addressed by the introduction of common source random variables, which are approximated by a Dunnett-Sobel class of variables [13]. In this framework, the standardized safety factor  $Z_i$  of the damage of a given component  $i$  is then represented as follows:

$$Z_i = \sqrt{1-r_i^2} \cdot V_i + r_i \cdot U \quad (1)$$

where  $V_i$  and  $U$  are standard normal random variables and  $r_i$  is a coefficient that is optimized in order to approximate the correlation factor  $\rho_{ij}$  between  $Z_i$  and the other safety factor variables:

$$\rho_{ij} \approx r_i \cdot r_j \quad \forall i, j \in [1, n] | i \neq j \quad (2)$$

For each combination of sampled values  $\{im; u; v_i\}$  from the variables  $IM$ ,  $U$  and  $V_i$ , the damage event of component  $i$  is finally expressed as the following condition:

$$\left( z_i = \sqrt{1-r_i^2} \cdot v_i + r_i \cdot u \right) \geq -\frac{\log im - \log \alpha_i}{\beta_i} \quad (3)$$

where  $\alpha_i$  and  $\beta_i$  are the mean and standard deviation for the fragility curve corresponding to component  $i$  exceeding the given damage state.

A BN formulation similar to the one presented in Gehl and D'Ayala [6] is adopted here, except that a layer of functionality nodes has been added in order to represent the aggregated functional consequences at the bridge level. The BN is comprised by the following nodes, for a bridge with  $n$  structural components (see Figure 2):

- A root node  $IM$  representing the seismic intensity applied to the  $n$  components.
- A root node  $U$  representing the standard normal variable that is common to all components.
- Root nodes  $V_1 \dots V_n$  representing the standard normal variables that are specific to each component.
- Nodes  $C_1 \dots C_n$  representing the component failure events: the conditional probability table (CPT) is built by following the condition in Eq. 3, for each combination of values that are generated by the parent nodes  $IM$ ,  $U$  and  $V_i$ .
- Nodes  $Du_1 \dots Du_n$  (resp.  $Fl_1 \dots Fl_n$ ) representing the repair duration (resp. functional loss) events: the CPT is generated by following the loss models that are detailed in Figure 1, i.e. the probability of reaching discretized levels of losses is directly read from the cdf plots.
- Nodes  $I_1 \dots I_k$  representing intermediate events: these nodes are used to build a chain structure, which is more robust than a naïve formulation, in terms of computation loads [14]. The rule assumed here is that the functional losses are sequentially built up by considering the maximum loss value when each component is added to the chain.
- Nodes  $S1$  and  $S2$  representing the final loss events for the two metrics, which take into account the contributions from all components: the CPT structure is the same as the one from the intermediate nodes.

- A node SYS representing the joint occurrence of the different values of repair duration and functional loss. This final node is essential for the computation of joint probabilities when the Bayesian analysis is performed.

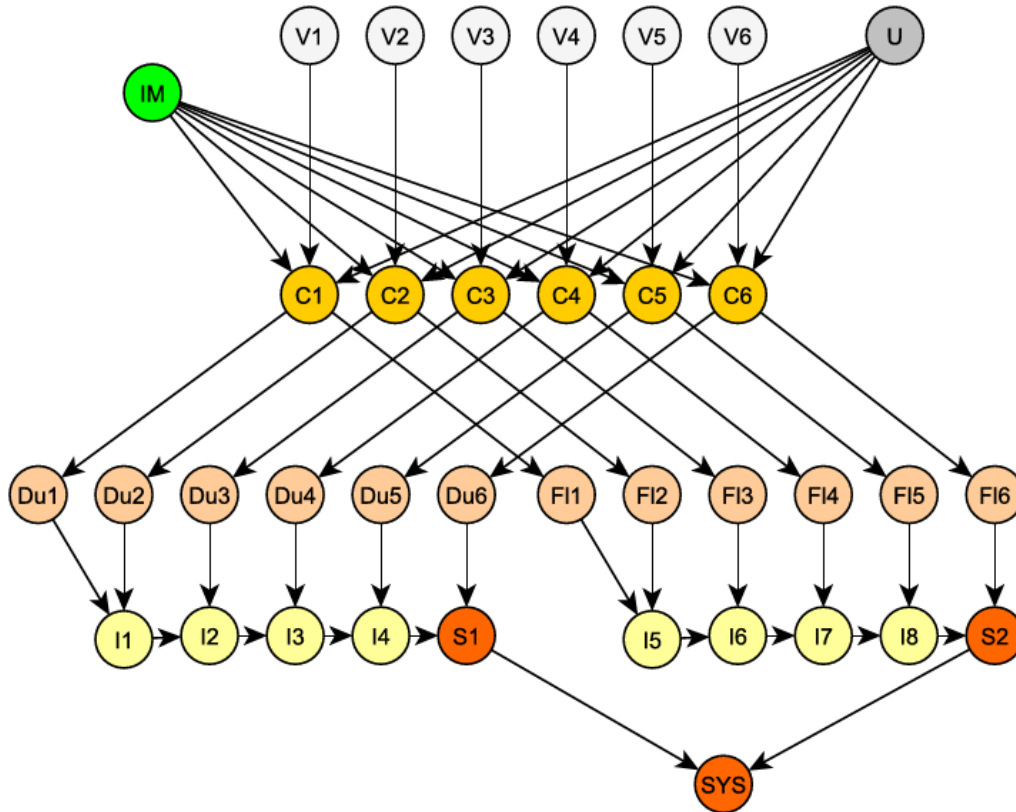


Fig. 2 – BN formulation for the quantification of functional consequences, for a bridge system with 6 structural components

This BN structure is then implemented in the Bayes Net toolbox [15], where exact inference can be performed through a junction-tree algorithm. First, a forward analysis may be performed by setting a given intensity measure (IM) and observing the updated distribution of repair durations and functional losses at the final node SYS. For successive values of IM, probabilistic functionality curves can then be derived point by point, as it will be demonstrated in the following section. On the other hand, the BN formulation enables also a backward analysis (i.e. Bayesian inference), which is useful to identify which components are contributing the most to a given functionality loss level.

### 3. Application to a bridge system

The aforementioned BN approach is demonstrated through a single bridge that is typical of reinforced concrete highway bridges.

#### 3.1 Bridge model

The multi-span simply-supported concrete (MSSSC) girder bridge model described by Nielson [8] is used as a case-study, since the model specifications are well detailed and this general typology corresponds to bridge types that are commonly found along Italian highways [7]. The studied structure is composed of three independent deck spans supported by three-column piers, while elastomeric bearings with steel dowels ensure the connections between the pier caps and the decks (see Figure 3). There is an alternation of fixed (grey circles) and

expansion (white circles) bearings. All the structural and mechanical parameters used here are taken from the MSSSC model [8].

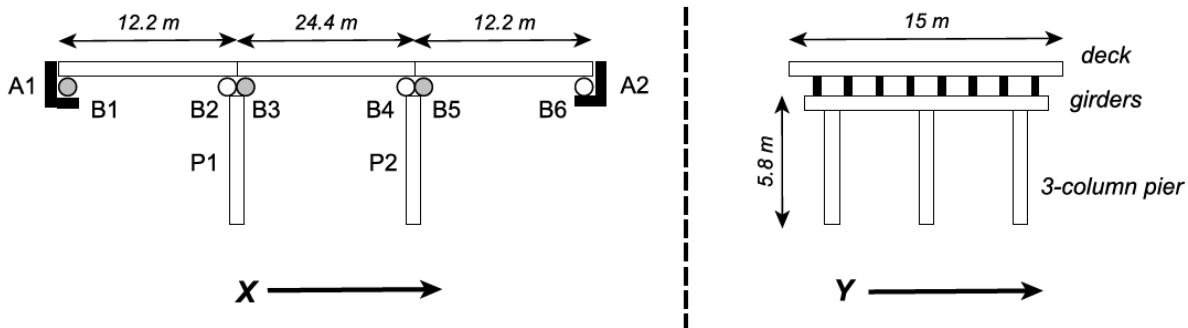


Fig. 3 – Bridge layout in longitudinal (left) and in transversal (right) directions

Ten vulnerable components are identified, i.e. two piers (P1 and P2), two abutments (A1 and A2) and six bearings (B1 to B6). The bridge has been modelled in the OpenSees finite element code [16], where preliminary pushover analyses are conducted in order to identify the limit states that correspond to the failure modes that have been proposed in Table 1. The selected limit states are detailed in Table 2 for both loading directions: shear failure of piers has not been considered, since the structural analysis has demonstrated that the piers are damaged due to flexural behaviour.

Table 2 – Proposed limit states for the selected component failure modes along both loading directions

ID	Component	Failure mode	Engineering Demand Parameter	Damage ‘Severity’							
				X-direction				Y-direction			
				D1	D2	D3	D4	D1	D2	D3	D4
1	Pier	Bending	Section curvature [1/m]	0.005	0.008	0.014	0.020	0.015	0.024	0.041	0.061
3	Abutment	Piles (active)	Deformation in tension [mm]	7.6	25.4	200.0	-	7.6	25.4	200.0	-
4	Abutment	Backfill (passive)	Deformation in compression [mm]	19.2	25.4	-	192.0	-	-	-	-
5	Fixed bearing	-	Deformation [mm]	10.5	10.5	12.5	152.0	10.5	10.5	12.5	406.0
	Expansion bearing			10.5	25.0	34.5	152.0	10.5	10.5	12.5	406.0

### 3.2 Component fragility curves

Fragility curves for all components and failure modes are derived by successively applying a set of ground motions along longitudinal and transversal directions (i.e. non-linear time-history analysis). The generation of the ground motion suite follows the magnitude-distance criteria that are prescribed by Nielson [8], in order to be consistent with the seismotectonic context of the area where the bridge has been modelled, i.e. Mw between 5.5 and 7.5 and epicentral distance between 10 and 100 km. Due to the lack of sufficient natural records, it is proposed to generate synthetic records using a stochastic procedure developed by Pousse et al. [17], which is based on the definition of a magnitude, an epicentral distance and an EC8 soil class. A total of 288 ground motions, spanning EC8 classes A to D, are then applied along each loading direction.



The statistical derivation of fragility curves uses the Generalized Linear Model (GLM) regression with the probit as the link function. Such an assumption enables to directly associate the regression coefficients with the mean and standard deviation of the cumulative lognormal distribution. The component fragility curves are then presented in Figure 4.

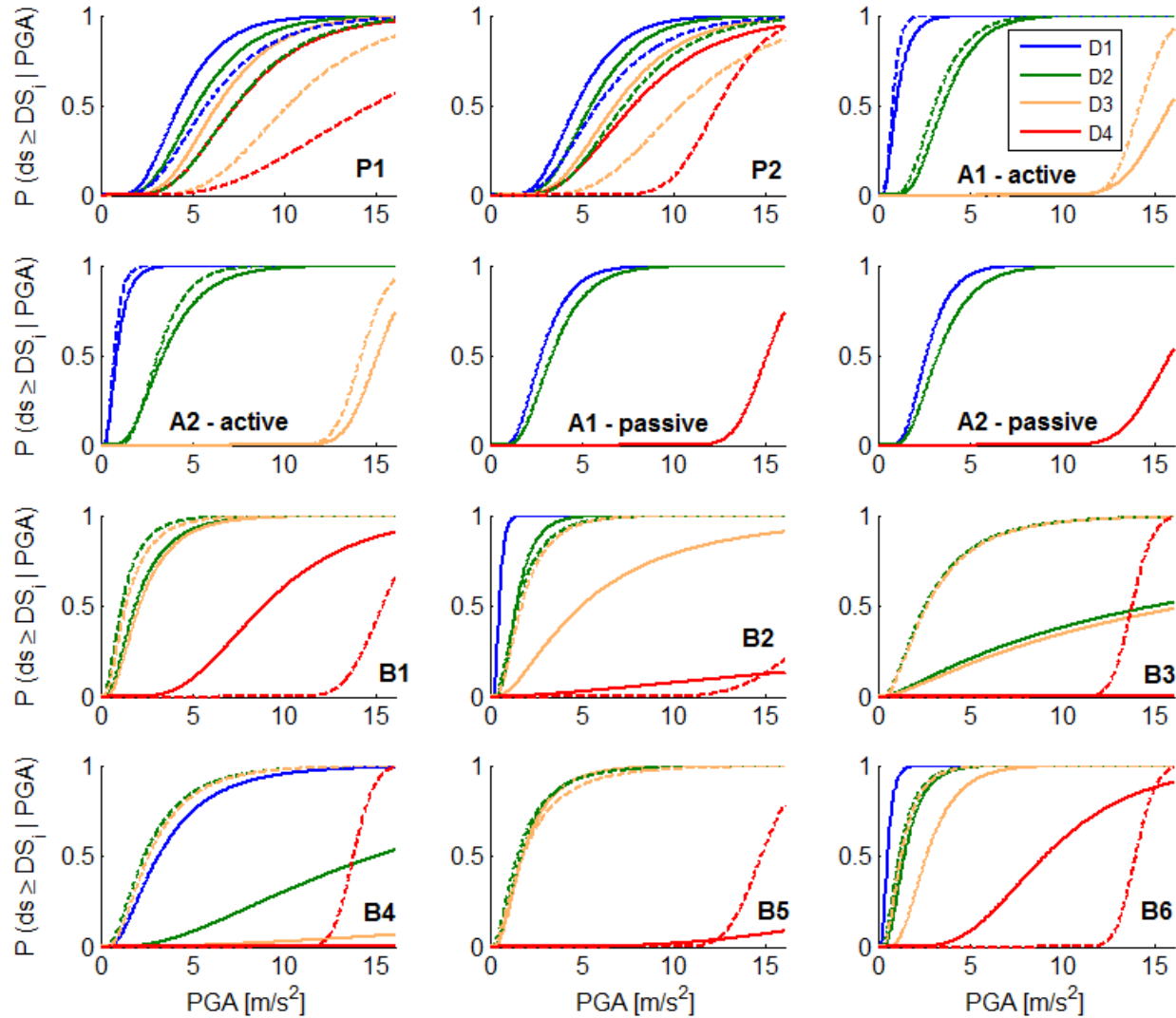


Fig. 4 – Component fragility curves for the bridge components in the X (solid lines) and Y (dashed lines) loading directions

The component fragility curves provide a first indication on which components are the most vulnerable and, therefore, the most likely to contribute to the functional losses. For instance, the component failure modes that are the most likely to occur at lower intensity levels are the minor cracking of abutment piles (i.e. active behaviour) and the deformation of bearings at the end spans. On the other hand, piers and bearings at the middle span appear to be less vulnerable in this specific bridge configuration.

The component responses for each simulation are also used to build the correlation matrix between the different damage events, as suggested by Song and Kang [11]: the responses are converted into standardized safety factors, for which the correlation matrix is approximated by a Dunnett-Sobel structure (see Eq. 1). This step is essential to account for the statistical dependence between components, which has significant influence on the global response. For instance, in the case of a series system, the global survival probability will be bounded by the product of all component survival probabilities on one hand (i.e. assumption of total

independence between components), and by the survival probability of the most vulnerable component on the other hand (i.e. assumption of full correlation between components).

### 3.3 Probabilistic functionality curves

Once the component fragility curves are derived and the Dunnett-Sobel variables are identified, the BN can be assembled according to the formulation presented in Figure 2. For the considered bridge example, 22 components are considered (i.e. 12 components in *X*-direction, 10 in *Y*-direction), resulting in a global BN containing 133 nodes and 196 edges. During the initialisation of the junction tree, the largest generated clique potential is comprised of 10,450,000 elements, which still leads to reasonable computation times (i.e. one inference operation is performed in around 2.6s on an Intel® Core™ i7 processor with 16 GB RAM). Discrete levels of functional consequences are then selected, so that the Bayesian analysis may directly quantify the probability of exceeding these loss levels given the seismic intensity, as shown in Figure 5.

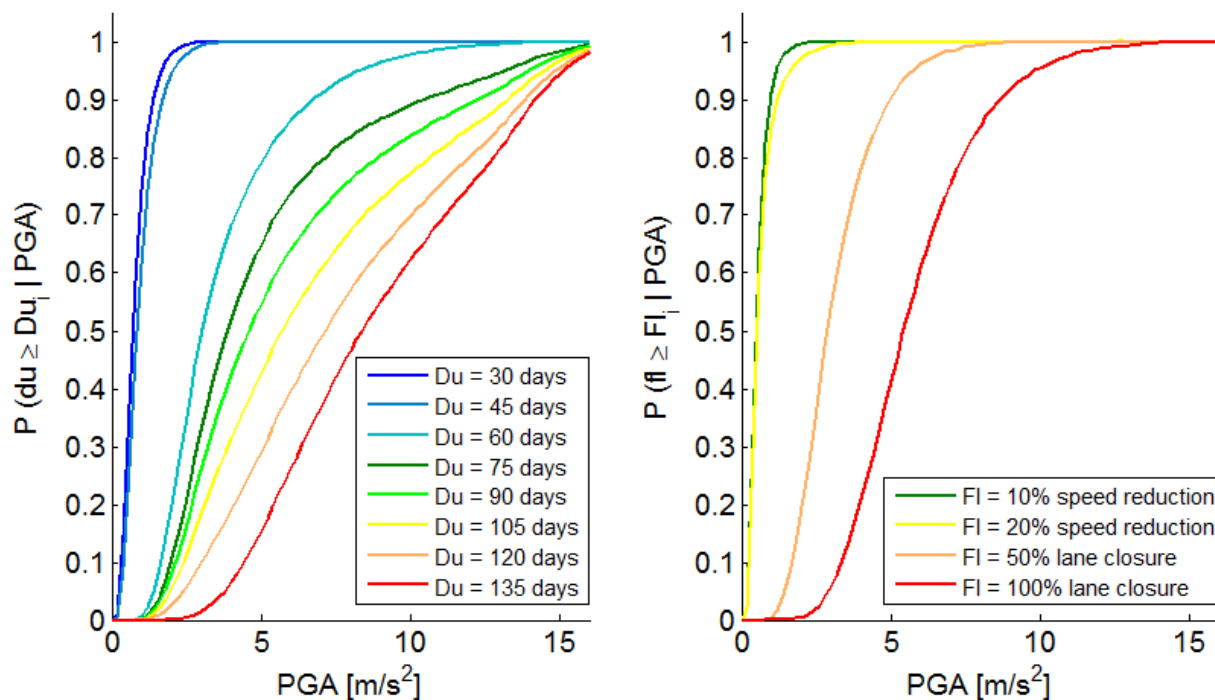


Fig. 5 – Probabilistic functionality curves for repair duration (left) and functional loss (right)

As expected, these probabilistic functionality curves reveal that the losses increase with the seismic intensities. Minor losses (e.g. short repair times or slight traffic disruption) appear to be induced by really low intensity levels, which is mostly due to the abutment piles reaching damage very quickly (i.e. see Figure 4). It should also be noted that the composition of various fragility curves does not necessarily lead to cumulative lognormal distributions for the functionality curves, especially for the repair duration curves: this observation prevents the use of simple statistical parameters to represent these curves, which have then to be expressed as tabulated values.

Finally, the curves in Figure 5 provide the marginal distributions for each loss metric taken separately, while the application of restoration strategies for a road network would require the joint knowledge of both the functional state and repair time of the elements at risk. The BN has been formulated in such a way that the repair duration node (i.e. node S1 in Figure 4) and the functional loss node (i.e. node S2) belong to the same clique during the generation of the junction tree. Therefore it becomes possible to quantify the joint probability of occurrence of the two loss metrics, as shown in Figure 6. This result is fundamental for the evaluation of the performance of the global road network, since such a loss representation enables the sampling of various consistent loss scenarios (e.g. for  $PGA = 3 \text{ m/s}^2$ , probability of 0.346 of experiencing 20% speed reduction for 45

days, probability of 0.075 of experiencing 50% lane closure for 120 days, etc.). There is some degree of correlation between the two loss metrics (i.e. the more severe the loss, the longer the repair time), even though some component failure modes may lead to complete closure while requiring reasonable restoration times (see Figure 1): therefore this aspect is able to be perfectly captured by the representation of the joint distribution of both metrics, which is easily accessed through Bayesian analysis.

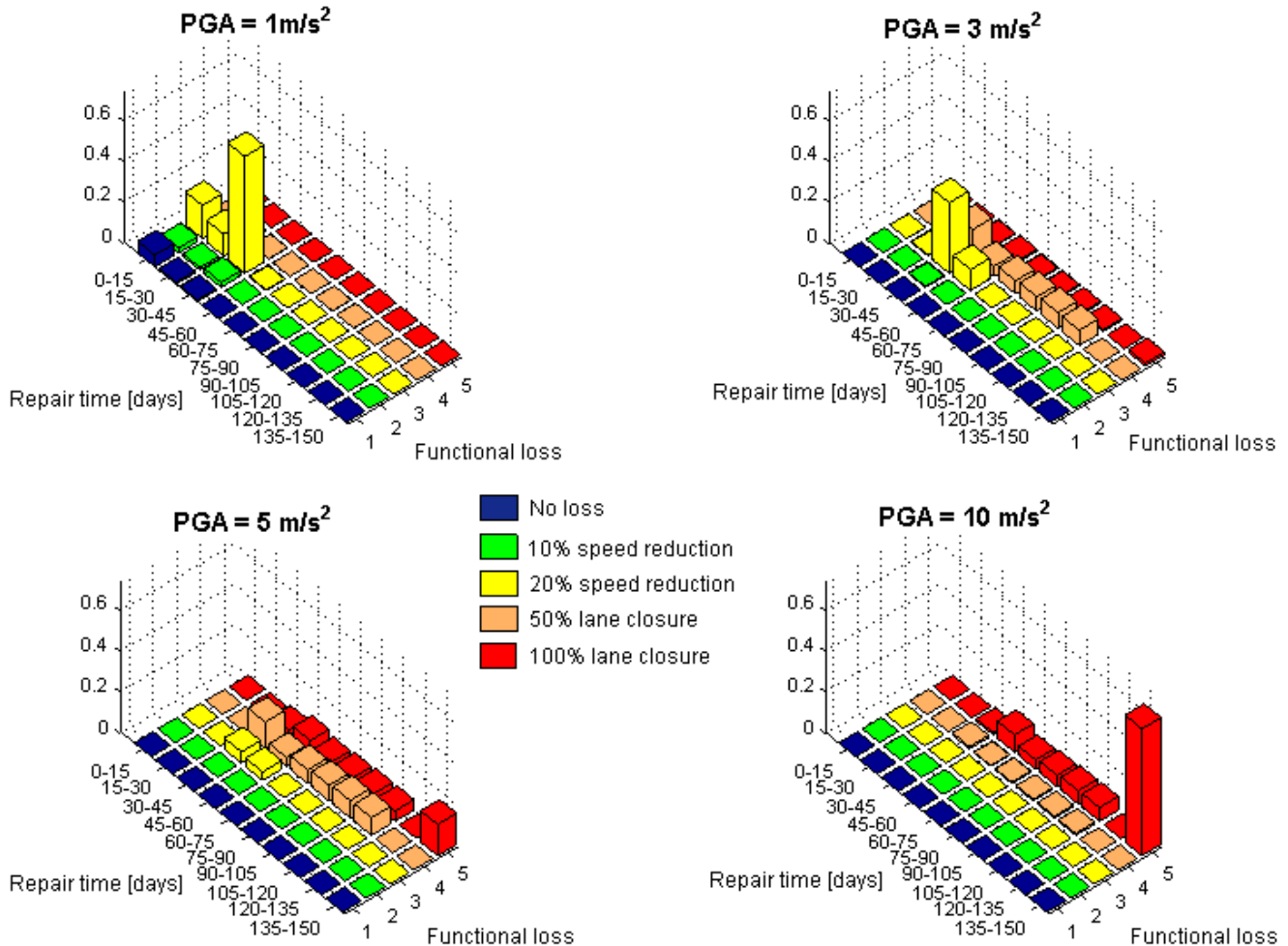


Fig. 6 – Joint distribution of the repair time and functional loss for different seismic intensity levels

### 3.4 Bayesian inference

Aside from deriving probabilistic functionality curves, the proposed BN formulation can also be used to quantify the respective role of each component in the functional state of the bridge system. Bayesian inference offers indeed endless possibilities in terms of assuming evidence at given nodes and observing the updated probabilities at other locations of the BN. For instance, one may assume full closure of the bridge for the longest duration (i.e. > 135 days): exact inference through a junction-tree algorithm can then update the conditional probabilities of occurrence of the other nodes' states.

As seen on Figure 7, the relative contribution of the bridge components to the bridge's full closure can be visualised for various levels of seismic intensity. Such information is essential in order to prioritize specific retrofit measures, which may be applied to different components depending on the seismicity level that is expected. In the present example, substructure components such as piers are contributing the most to bridge closure, which is due to both their high impact on functional losses (see Figure 1) and their relatively high fragility in the X-direction (see Figure 4).

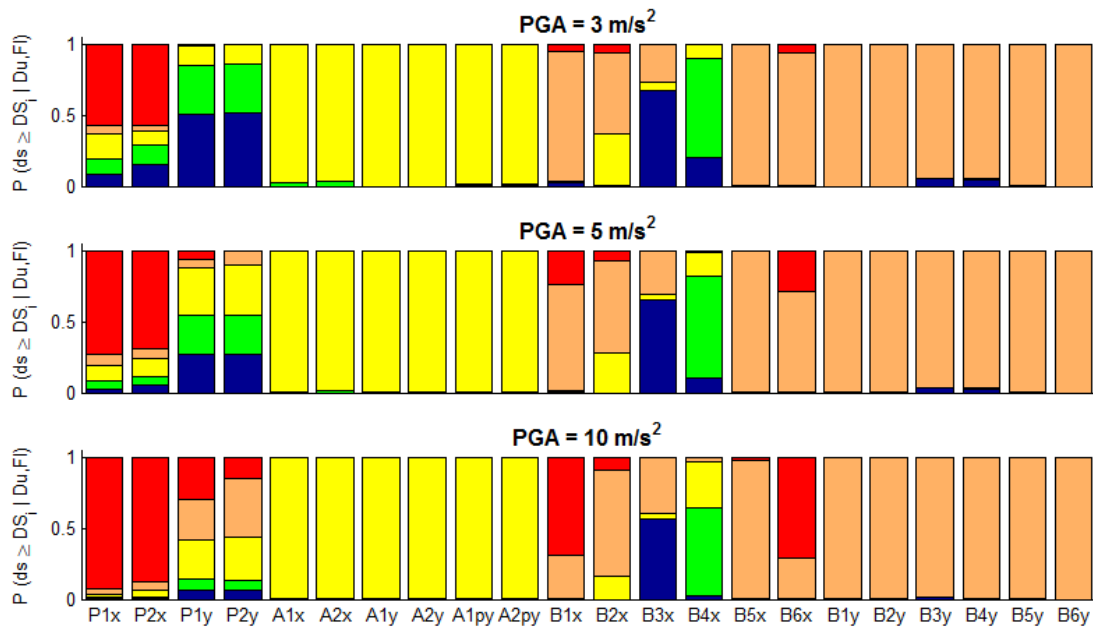


Fig. 7 – Distribution of component damage states given total functionality losses ( $F_i = 100\%$ ,  $D_u > 135$  days) and various seismic intensity. Damage states DS0 to DS4 are respectively represented by blue, green, yellow, orange and red bars.

#### 4. Conclusions

This paper has demonstrated the derivation of probabilistic functionality curves, which are similar to fragility curves, except that they directly express functionality losses instead of physical damage states. Therefore such a formulation is essential to deliver loss measures that are directly relevant to the performance assessment of infrastructure systems. For instance, in the case of road networks, expressing the damage to bridges in terms of repair duration and lane closure or speed reduction enables the application of traffic models on a degraded system (i.e. lane closure and speed reduction are directly linked to the flow capacity of a given road segment) and the optimization of restoration strategies (i.e. prioritization of repair operations based on duration and impact on traffic).

The use of BNs for the aggregation of component fragility curves is essential in order to compute the probability of occurrence of various functionality levels. The characteristics of the proposed BN structure account for the statistical dependence between component damage states (i.e. inter-component correlation), as well as for the joint probability of occurrence of various loss metrics (i.e. repair duration and functional loss). Finally, Bayesian inference allows for a backward analysis, which is useful to track the respective contribution of the components to the global losses.

However, it should be noted that the loss models used here (see Figure 1) are very coarse and should be considered for demonstration purposes only. Therefore further involvement with infrastructure managers is highly recommended in order to refine these models. Another assumption to be investigated is the aggregation of losses from component level to bridge level: currently the maximum value is considered, while a summation of losses could also be adopted, depending on whether multiple component failures on the same bridge have a cumulating effect.

#### 5. Acknowledgements

This study has been carried out within the INFRARISK project (Novel indicators for identifying critical INFRAstructure at RISK from Natural Hazards), which has received funding from the European Union's



Seventh Programme for research, technological development and demonstration under grant agreement No 603960.

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