Analysis of a pumping test in a horizontal fracture

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ANALYSIS OF PUMPING TESTS PERFORMED IN A HORIZONTAL RECTANGULAR FRACTURE

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ABSTRACT

The analytical solution has been derived for a constant rate uniform flux pumping test in a single horizontal rectangular fracture. The solution is given for the drawdown at the pumped well and at an observation well inside or outside the fracture. A set of type curves which depend on two dimensionless parameters has been drawn. These type curves give the dimensionless drawdown at the pumped well versus the dimensionless time according to the shape factor of the fracture and the dimensionless aquifer thickness. Another set gives the average drawdown along the vertical at an observation well versus dimensionless time depending on the dimensionless distance and the shape factor.

For both the drawdown at the pumped well and at an observation well the solution has also been studied for early times and long times in order to derive approximative solutions which are compared to the well known solutions in isotropic aquifers.

1. INTRODUCTION

This paper presents results which have been obtained during the research contract n° 563-78 EGF for the Commission des Communautés Européennes. These results are described in detail , in French, in BERTRAND L., FEUGA B., NOYER M.L., THIERY D. 1980[1].

A review of the litterature has shown that analytical solutions have been derived to determine the unsteady state pressure distribution created by a well with a single flat fracture.

The available solutions are the following :

- a vertical finite length fracture (Gringarten et al, 1974)[2]
- a horizontal finite radius circular fracture (Gringarten et al,1974)[3]
- an inclined fracture (Cinco et al,1975)[4]
- a vertical fracture partial penetration (Raghavan et al,1976)[5]

For all these schemes (except the vertical fracture) only the pressure at the pumped well has been computed. These schemes are interesting but they are not always adapted to real fractures mainly because of anisotropy of permeability in the horizontal directions.

A horizontal fracture is more likely to be elliptical than circular. In order to analyse drawdowns in such a fracture we derived analytical solutions for a pumping (or injection) test in a rectangular fracture which is a closer approximation to an elliptical fracture.

The drawdown has been calculated at the center of the fracture but also at an observation well.

2. DESCRIPTION OF THE SYSTEM

The geometry of the system is defined as follow :

the aquifer has an infinite lateral extension; its characteristics are :

- constant thickness h
- constant storage coefficient S

  (specific storage coefficient $S_s = S/h$)
- anisotropic permeability : Kx, Ky, Kz

The fracture , which is situated in the middle of the aquifer, has a rectangular shape with :

- a length 2 xf
- a width 2 yf
- a negligible thickness

3. BOUNDARY CONDITION INSIDE THE FRACTURE

Two types of boundary conditions may be set inside the fracture :

- an infinite conductivity fracture
- a uniform flux fracture.

a) An infinite conductivity fracture

It is a fracture which has an equivalent hydraulic conductivity (or permeability) much higher than the conductivity of the surrounding formation. As a matter of fact its transmissivity (K.h product) is much higher then the rock transmissivity This condition is usually met in thick fractures in a low permeability formation. The head gradient in an infinite conductivity fracture is equal to zero.

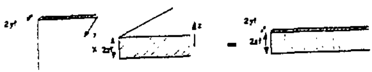
b) A uniform flux fracture

The discharge from the aquifer to the fracture is performed at a uniform rate per unit area of fracture.

This boundary condition is probably not exactly fulfilled but, as will be shown later, it is a close approximation of the infinite conductivity boundary condition.

The uniform flux boundary condition is very interesting because it makes it possible to compute the drawdowns analytically by the "source functions method" . This method has been described by(GRINGARTEN A.C. and RAMEY H.J. 1973)[6].

The source function associated to the rectangular fracture is obtained as the product of 3 elementary source functions as shown below.



The product is then simplified considering that the thickness zf of the fracture is negligible compared to the thickness h of the aquifer.

c) Dimensionless notations

The following dimensionless variables are used :

- coordinates :

$x_D = x/xf$

$y_D = y/yf$

$z_D = z/h$

- drawdown

$s_D = 4\pi Ts/Q$ with $T = h\sqrt{KxKy}$ = transmissivity

- geometry factors

$F = (xf/yf)\sqrt{K_y/K_x}$ (shape factor)

$h_D = (h/\sqrt{x_f y_f}) \cdot \sqrt{K/Kz}$ (dimensionless thickness of the aquifer)

- time

$t_{DF} = \pi Tt/x_f y_f S$ (relative to the fracture area)

$t_{DX} = 4\,K_x ht/x^2 S$ (relative to the distance to the center of the fracture).

Nota : All these notations are consistent with Theis's notations.

The shape factor F is the ratio of the length of the fracture to its width (corrected by the horizontal anisotropy factor)

For an horizontally isotropic aquifer :

F = 1 applies to a square fracture (it is the shape which is the closest to the circular fracture)

F = 10 applies to a rectangular fracture with a length equal to 10 times its width.

The dimensionless thickness h_D is the ratio of the aquifer width to the fracture extension (corrected by the vertical anisotropy factor) .The fracture extension is the geometrical mean of its length and its width.

For a fully isotropic aquifer :

$h_D = 0.1$ applies to a fracture with an extension equal to 10 times the thickness of the aquifer

$h_D = 10$ applies to an aquifer of thickness equal to 10 times the extension of the fracture.

4. DRAWDOWN AT THE CENTER OF THE FRACTURE

The drawdown at the center of the fracture is given by the following expressions which are easy to compute numerically :

$$s_D = \int_0^{t_{DF}} \mathrm{erf}\sqrt{\frac{\pi F}{4\tau}} \cdot \mathrm{erf}\sqrt{\frac{\pi}{4F\tau}} \times \quad \ldots$$

$$\ldots \left[1 + 2\sum_{p=1}^{\infty} \exp\left(-\frac{4\pi p^2 \tau}{h_D^2}\right)\right] d\tau \qquad (1)$$

or :

$$s_D = \int_0^{t_{DF}} \mathrm{erf}\sqrt{\frac{\pi F}{4\tau}} \cdot \mathrm{erf}\sqrt{\frac{\pi}{4F\tau}} \cdot \frac{h_D}{2\sqrt{\tau}} \times \ldots$$

$$\ldots \left[1+2\sum_{p=1}^{\infty} \exp\left(-\frac{\pi p^2 h^2{}_D}{4\tau}\right)\right] d\tau \qquad (2)$$

Expression (1) is easier to compute for late values of dimensionless time and expression (2) for early values.

The calculations have been performed for the following values of the geometry factors :

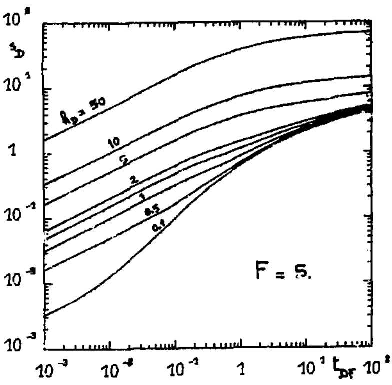
h_D = 0.1, 0.5, 1, 1.5, 2, 5, 10, 50.

and :

F = 1, 2, 5, 10, 50 which corresponds also to 1. 0.5, 0.2 0.1, 0 02 .

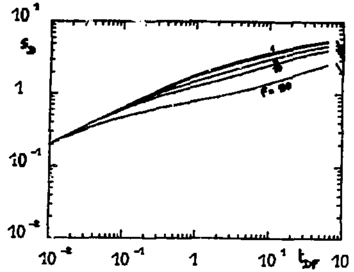
Two kinds of type curves have been drawn :

- type curves for constant shape factor F (see graph 1)



Graph 1 - Drawdown at the center of a horizontal rectangular fracture

- type curves for constant dimensionless thickness h_D (see graph 2)



Graph 2 - Drawdown at the center of a horizontal rectangular fracture

For early dimensionless times the curves show a typical "one half slope" of equation :

$$s_D = h_D \sqrt{t_{DF}} \qquad (3)$$

This equation corresponds to a vertical flow from the formation to the fracture. It does not depend on the shape factor F because there is no flow in the horizontal directions.

For late dimensionless times the drawdown is a linear function of the logarithm of the time according to the following equation :

$$s_D = s_{DO} + L_n \frac{t_{DF}}{t_{DFO}} \qquad (4)$$

(s_{DO} is the drawdown at time t_{DFO})

This equation shows that, after a time t_{DFO} the variation of the drawdown does not depend on the geometrical factors F and h_D any more.

It is a radial flow which agrees with JACOB's approximation. The drawdown variation is plotted as a straight line on semi-logarithmic paper but Theis curve can not be recognized on log-log paper because of the constant s_{DO}

Equation (4) is valid after the following dimensionless times t_{DF} corresponding to a maximum error of 10% :

F \ h_D	0.1	1	10	100
1	5.2	5.2	23.8	2384
2	6.5	6.5	23.8	2384
5	13.6	13.6	23.8	2384
10	26.4	26.4	30	2384
50	131	131	131	2384

Equations (1) and (2) refer to the center of the fracture, but as a matter of fact, it is a close approximation of the drawdown at any location inside the fracture, because as it will be shown later the drawdown inside the fracture is nearly equal everywhere.

5. DRAWDOWN AT AN OBSERVATION WELL

A difficulty arises because the drawdown field is 3 dimensionnal : the drawdown is different at each altitude of an observation well. This difficulty has been solved considering a small diameter observation well with perforations along the whole aquifer thickness. The drawdown measured by such an observation well is the average of the drawdown along the vertical direction. This average , which has been computed analytically does not depend on the dimensionless thickness h_D.

The average drawdown at an observation well situated along the ox axis is given by the following expression :

$$s_D = \frac{3}{4} F x_D^2 \int_0^{t_{DX}} \frac{1}{2} \left[\operatorname{erf} \frac{\frac{1}{x_D}+1}{\sqrt{\tau}} + \operatorname{erf} \frac{\frac{1}{x_D}-1}{\sqrt{\tau}} \right] \cdots$$

$$\cdot \operatorname{erf} \frac{1}{x_D F \sqrt{\tau}} \, d\tau \qquad 5)$$

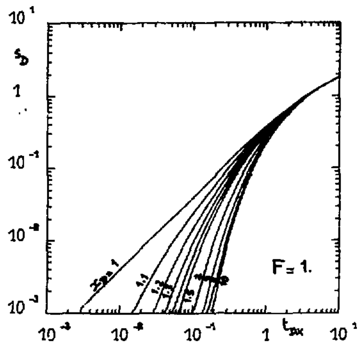
The drawdown at an observation well situated along the oy axis is given by the same expression after replacing F by 1/F.

Expression (5) has been calculated numerically for the following values of the parameters :

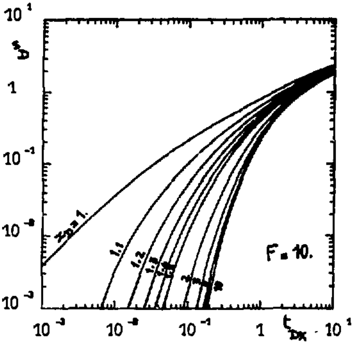
x_D = 1, 1.1, 1.2, 1.3, 1.4, 1.5, 2, 3, 10

F = 0.1, 0.5, 1, 2, 5, 10, 50

Two examples of type curves are drawn on graph 3 and 4. They show that for $F \geq 1$ (observation wells situated along the ox axis), the curves are very close to Theis's solution ($x_D = \infty$) as soon as $x_D \geq 2$ even for early dimensionless times. For late dimensionless times the flow is radial and is described by equation (4) following JACOB'S approximation .



Graph 3 - Drawdown at an observation well (horizontal rectangular fracture)



Graph 4 - Drawdown at an observation well (horizontal rectangular fracture)

Equation (4) is valid after the following dimensionless times corresponding to a maximum error of 10 %

F / X_D	1	2	5	10
1	16.7	14.2	13.5	13.4
2	11.7	11.0	10.9	10.8
5	10.3	10.2	10.1	10.1
10	10.1	10.	10.	10.

It appears that JACOB'S approximation applies as soon as the dimensionless time is greater than about 10 (as in a pumping test without any fracture, excepted for $x_D = 1$, i.e., at the contact of the fracture, where this minimal time is increased to 13 to 17.

6. DRAWDOWN AT AN OBSERVATION WELL IN CONTACT WITH THE FRACTURE

The average of the drawdown along the vertical has been computed because it doesn't depend on h_D any more. The calculations have been performed for various dimensionless times.

For early dimensionless times the drawdown is given by the following equations :

$$\begin{cases} s_D = t_{DF} & 0 \leqslant x_D < 1 \quad (6) \\ s_D = \dfrac{t_{DF}}{2} & x_D = 1 \quad (7) \end{cases}$$

The initial drawdown averaged along the vertical is linear with time ; it is represented by a straight line of slope 1 in log-log coordinates.

Equations (6) and (7) are valid, with a maximum error of 10% , before the following value of dimensionless time is reached :

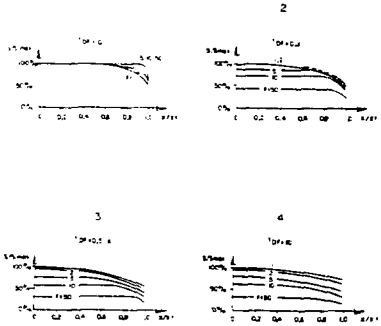
$$t_{DF} < 1/F \quad (8)$$

expression (8) shows that the linear drawdown is observed during a longer time when the shape factor is smaller i.e. when the shape of the fracture is closer to a square.

For late dimensionless times the variation of the drawdown is given by equation (4) . This equation , which describes JACOB's approximation, is valid as soon as the following dimensionless time is reached

X_D / F	0	0.2	0.6	1
1	5.2	5.6	8.1	13.1
2	6.5	7.2	12.2	22.3
5	12.6	15.2	27.8	53
10	26.4	29.6	54.7	105
50	131	147	272	[illegible]24

It appears that JACOB'S approximations is valid only after a long dimensionless time (greater than 100) for very rectangular fractures i.e. fractures with a large F shape factor. The drawdown inside the fracture has been drawn for different dimensionless times on graph 5. This graph shows that the drawdown is approximatively constant inside the fracture. It means that the equivalent transmissivity of the fracture is very high.



Graph 5 - Drawdown inside a horizontal rectangolar fracture (average along the aquifer width)

The drawdown corresponding to a pumping test performed in a infinite transmissivity horizontal rectangular fracture is then closely approximated by the formula that we have derived for a uniform flux horizontal rectangular fracture.

7. CONCLUSION

A new scheme has been derived for pumping tests in fractured aquifers. This scheme is characterized by a single horizontal flat rectangular fracture.

The drawdown has been computed at the center of the fracture and also at an observation well situated along the principal directions. Types curves have been drawn which makes it possible to perform pumping test analysis or to compute the drawdown corresponding to a known rectangular fracture.

NOTATIONS

x, y, z : coordinates relative to the center of the fracture

x_D, y_D, z_D : Dimensionless coordinate

x_f : Fracture half length

y_f : Fracture half width

t = Time

τ = Dummy variable representing time

t_{DX}, t_{DF} : Dimensionless time

h = Aquifer thickness

h_D = Aquifer dimensionless thickness

Q = Pumped or injected discharge

s = Drawdown (or head difference)

s_D = Dimensionless drawdown

K_x, K_y, K_z : Permeability (hydraulic conductivity)

$K = \sqrt{Kx\ Ky}$ = Horizontal permeability

T = Horizontal transmissivity (K.h product)

S = Storage coefficient

S_s = Specific storage coefficient = S/h

F = Fracture geometry factor

π = 3.14159 ...

erf = Error function

exp = Exponential function

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