Multiphase Modelling of a Gas Storage in Aquifer with Automatic Calibration and Confidence Limits

Dominique Thiéry, Karine Guedeney

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DOMINIQUE THIERY
BRGM Direction de la Recherche Laboratoire Géochimie et Hydrologie, B.P. 6009, 45060 Orléans Cedex 2, France
E-mail : d.thiery@brgm.fr

KARINE GUEDENEY
Gaz de France Direction de la Recherche, Département Production Stockage,361, avenue du Président Wilson, BP33, 93211 Saint-Denis La Plaine Cedex, France

Abstract Multiphase flow modelling involving gas and water is widely used in gas dissolution in aquifers or in aquifer gas storage. The parameters related to the gas are usually well known but the parameters of the aquifer system are not. In order to obtain reliable forecasts, it is necessary to calibrate the multiphase model on monitored data. This can be done by automatic calibration followed by the determination of the confidence limits of the parameters, and of the confidence limits of the forecasts. This paper presents a 3D-multiphase flow model for the simulation of Gas-Water, or NAPL-Water couples. It integrates automatic calibration procedures based on Marquardt's algorithm for all types of parameters: hydraulic conductivity, which is classical, and also multiphase flow constitutive relations, aquifer compressibility and boundary limits. The model may be calibrated simultaneously on observation of different kind of data e.g. gas pressure, hydraulic heads, water saturation etc. A sensitivity analysis method, which integrates spatial or temporal dependence of the observations, follows the automatic calibration. The model is applied to the data from an actual gas storage situated in a gas cap in an aquifer near Paris (France) monitored by Gaz de France national company during more than 30 years. The automatic calibration enabled to determine the hydraulic conductivity, the rock compressibility and the lateral boundary limits. It resulted in a very accurate simulation of the gas pressure resulting from the periodical injections and withdrawals of gas. The sensitivity analysis enabled to determine the parameters that were the most important.

Notations

A	=	Exchange area between cells	$[L^2]$	Vol	=	Volume of a cell	$[L^3]$
hc	=	Suction = hg – hw	$[L]$	hg	=	Gas hydraulic pressure head	$[L]$
hw	=	Water hydraulic pressure head	$[L]$	H_w	=	Water hydraulic head = hw + z	$[L]$
STO_g	=	Stored mass of gas	$[M]$	STO_w	=	Stored volume of water	$[L^3]$
x, dx	=	Distance between two cells	$[L]$	t , dt	=	Time , Time step	$[T]$
z	=	Elevation	$[L]$	k	=	Permeability	$[L^2]$
K_{rw}	=	Water relative conductivity	$[-]$	K_{rg}	=	Gas relative conductivity	$[-]$
β_w	=	Water compressibility	$[L^{-1}]$	β_{por}	=	Pores compressibility	$[L^{-1}]$
θ_g	=	Volumetric gas content	$[L^3L^{-3}]$	θ_w	=	Volumetric Water content	$[L^3L^{-3}]$
μ_g	=	Gas viscosity	$[ML^{-1}T^{-1}]$	μ_w	=	Water viscosity	$[ML^{-1}T^{-1}]$
ρ_g	=	Gas density	$[ML^{-3}]$	ρ_w	=	Water density	$[ML^{-3}]$
ω	=	Porosity = $\theta_w + \theta_g$	$[L^3L^{-3}]$	d_g	=	Gas relative density = ρ_g / ρ_w	$[-]$
Q_w	=	Volumetric flow of water	$[L^3T^{-1}]$	Q_g	=	Mass flow of gas	$[MT^{-1}]$
g	=	Gravity acceleration	$[LT^{-2}]$				

INTRODUCTION

Depending on the season, the consumption of natural gas undergoes significant fluctuations. The fact that sources supply tend to be remote complicates the task of adapting deliveries to seasonal and daily variations in demand. Early on, the need thus arose to constitute natural gas reserves. As far back as 1954, Gaz de France began developing techniques for storing gas underground. Underground storage facilities help balance out the sharp contrasts in annual demand between winter and summer. They also serve to meet peaks in demand on the coldest winter days. Moreover, they have to be ready to deal with exceptional situations like the record cold spells that occur statistically twice in every century. Last, they compensate for the possible failure of a supply source. The inventory reflects the evolution of the demand that has risen steadily and especially in winter.

To answer this demand, attention is to be focused both on the quality of the data, in order to have a very precise knowledge of the storage at all times, and on the quality of the reservoir modelling tool that simulates the storage behaviour and forecasts its future performance. For the forecasts to be valid, a numerical simulation should first adequately reproduce the past performance of the reservoir. The task of finding a reservoir description yielding a correct simulation of the past performance of the reservoir is called history matching (HM). For the HM to be correct, we need to have accurate measured pressure, appropriate reservoir modelling tool and suitable HM technique by automatic calibration. The difference between simulated outputs and measured data, i.e. the data residuals, will then be acceptably small, and the performance forecasts reliable.

MULTIPHASE MODELLING OF FLOWS

Equations to be solved

At each time step it is necessary to solve a system of non linear equations resulting from (1) the volume conservation of water in each cell, (2) the mass conservation of gas in each cell and (3) the equations of state for gas and water. These equations, expressed in finite volume, are :

Volume conservation of water (subscript *w*) :

$$\sum_i T_{wi}\left(H_i - H_w\right) + Q_w = \frac{d\left(STO_w\right)}{dt}$$

Mass conservation of gas (subscript *g*) :

$$\sum_i T_{g_i}\rho_{gi}\left[\left(h_{g_i} - h_g\right) + d_{gi}\left(z_i - z\right)\right] + Q_g = \frac{d\left(STOg\right)}{dt}$$

With :

p = Index for the beginning of the time step

i = Index of an adjacent cell

ρ_{gi} = Weighted gas density with adjacent cell *i*

d_{gi} = Av. relat. density with cell *i*

$T_{wi} = k\ \rho_w / \mu_w\ K_{rw}\ A / dx$

$T_{gi} = k\ (\rho_w / \mu_g)\ K_{rg}\ A / dx$

$STO_w = Vol\ [(\theta_w - \theta_{wp})] + \theta_w\ (\beta_w + \beta_{por})(H_w - H_{wp})]$

$STO_g = Vol\ [(\theta_g \rho_g - \theta_{gp} \rho_{gp}) + \rho_g \theta_g \beta_{por} (h_g - h_{gp})]$

Resolution of the system of equations

The equations are solved with the 3D finite volume code MARTHE described by Thiéry (1990, 1993b). Three formulations are available for the unknowns : (water head - volumetric content) or (Gas pressure - volumetric content) or (water head - gas pressure). Picard iterations are used to take care of the non-linearities. The transfer coefficient between two adjacent cells uses an upstream weighting of the adjacent relative conductivities (i.e. it is the relative conductivity of the upstream cell, which is used). This scheme has proved to be very stable and efficient. For the present case a "water head - gas content" scheme has been chosen. In each cell the unknowns are the head of the water phase *Hw* and the gas content θg. The volume and mass conservation equations are written with these variables converting θw to θg using the relation $\omega = \theta w + \theta g$ and converting *hg* to *Hw* using the suction h_c definition. Then θg is eliminated by adding the 2nd equation to the 1st one multiplied by ρg. One obtains a system of *n* equations, *n* being the number of cells, with the only variable *Hw* which is solved classically. From the computed values of *Hw* the variables θw are immediately obtained explicitly in each cell from equation 1. Then in each cell *hg* is obtained immediately from suction definition and θg from $\omega = \theta w + \theta g$.

Geometry of the model, boundary conditions and initial parameters

The underground gas storage of Saint-Illiers-la-Ville is located 50 km west of Paris (France) in the Upper Oxfordian. It is a fairly radial sandstone reservoir 30 m thick situated 340 m deep below sea level. The aquifer gas storage now presents a central gas cap surrounded by an active aquifer. The maximum inventory is about 1500 millions of normal cubic meters (MMm3 (n)) and the working gas volume is roughly 650 MMm3 (n). Figure 1 shows a top view of the reservoir.

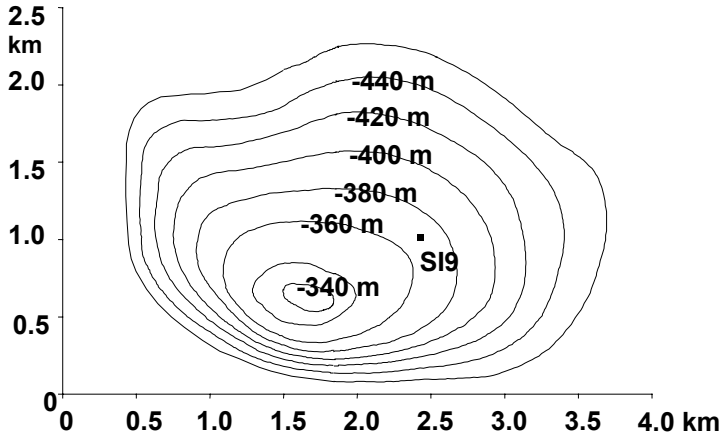

Fig. 1 Top view of Saint-Illiers reservoir: isobathes and observation well.
The production wells (not shown here) are located at the top of the reservoir.

The reservoir has been modelled in radial coordinates. The grid is as follow : a circular cell of radius 500 m, where gas injection or withdrawal take place, surrounded by 22 ring cells 50 m wide. Then larger ring cells extend the domain to a radius of 82 km. (See Ory *et al*. 1997). The vertical thickness of each ring cell is 30 m and the top of the formation is decreasing from -340 m (below msl) in the centre to –460 m at a distance of 1.7 km. In each cell the initial

value of permeability k has been fixed to 2 Darcys (1.97×10^{-12} m^2) and the initial value of the porosity ω to 28 %. The compressibility of the pores β_{por} has been fixed to 2×10^{-5} m^{-1} and the compressibility of the water β_w to 0.5×10^{-5} m^{-1}. The constitutive laws for the relative permeability, the capillary pressure (suction), the correction of the gas compressibility with the pressure (the natural gas is not a "perfect gas") and the gas viscosity have been provided by Gaz de France. The gas pressure is monitored at the head of the observation well SI9 that corresponds approximately to the 2nd cell of the model (525 m from the injection). The pressure history is corrected to a reference datum of 360 m below sea.

The gas rate of injection or withdrawal and the gas pressure measurements are available during a long period of 33 years extending from 1965 (when natural gas injections started) to 1997. 1656 pressure measurements are available which corresponds to weekly observations. At the beginning of the storage (1965) there is no gas and the water phase is immobile with a hydraulic head estimated to 141.11 m which corresponds to a pressure of 616.11 m (60.44 bars) at the outer limit where the mean altitude is –475 m. At the outer limit the hydraulic head is prescribed and the gas content is prescribed at 0. Considering the uncertainty of the hydraulic head at the outer limit, and the relative altitude where the gas pressure is monitored, a correction has been applied to this head with an initial value $H0$ = +30 m (+2.94 bars). The simulation with MARTHE code has been performed with automatic time steps prescribed to range in the interval [0.5 – 4 days]. On a classical PC computer (Pentium II) approximately 6 min. of CPU time are needed for a simulation of the total period.

AUTOMATIC CALIBRATION OF THE PARAMETERS

Marquardt algorithm (Marquardt, 1963; Press *et al*., 1992), which has been implemented in MARTHE code, has been used to calibrate four parameters : k, ω, β_{por} and $H0$. This algorithm, which is more efficient than the former method described by Thiéry (1993a, 1994), necessitates the computation of the gradients of the observations (the monitored pressure) with respect to each parameter in order to get a Hessian. The gradients have been computed by finite difference, which necessitates, for each iteration of the algorithm, one simulation per parameter plus a reference simulation. Anterion *et al*. (1989) present a faster method for the direct calculation of all the gradients in a single simulation which could be used.

Table 1 displays the evolution of the calibration of the four parameters. The standard deviation of the observations σ_{obs} is 72.05 m (7.07 bars) and the initial Root Mean Square of Error $RMSE$ is 2.53 bars. The calibration criterion is defined as $(RMSE / \sigma_{obs})^2$ which is the percentage of residual errors (unexplained variance). After two iterations the criterion is reduced from 12.8% to 0.96%. After four iterations the parameters are calibrated and the $RMSE$ is equal to 0.6 bar.

Table 1 Automatic calibration of parameters.

	Iteration :	0	1	2	3	4
Permeability	(D)	2	0.519	0.669	0.688	0.729
Porosity	(%)	28	31.3	29.6	24.5	22.45
Head correction	(m)	30	21.2	10.4	-2.78	-7.89
Pore Compressibility	(m^{-1})	2	7.82	2.68	1.36	0.857
Criterion	(%)	12.8	3.60	0.963	0.713	0.693
RMSE	(m)	25.83	13.66	7.07	6.09	5.99
RMSE	(bars)	2.53	1.34	0.69	0.597	0.588

Figure 2 displays the comparison of the monitored gas pressures to the simulated ones. As the simulation with MARTHE code is very accurate the two curves can hardly separated on

the graph. It should be noted that the low pressures due to a strong depletion in the winter of 1984-1985 and the peak resulting from a significant increases in maximum inventory in the summer of 1995 are well simulated. Figure 3 displays the initial and final series of pressure deviations.

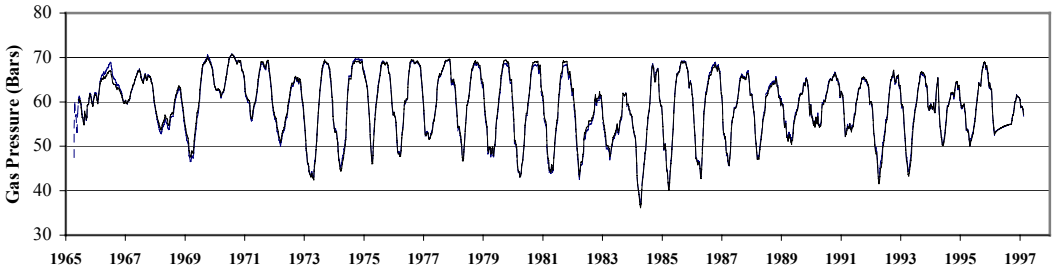

Fig. **2** Monitored and simulated gas pressure at the observation well.

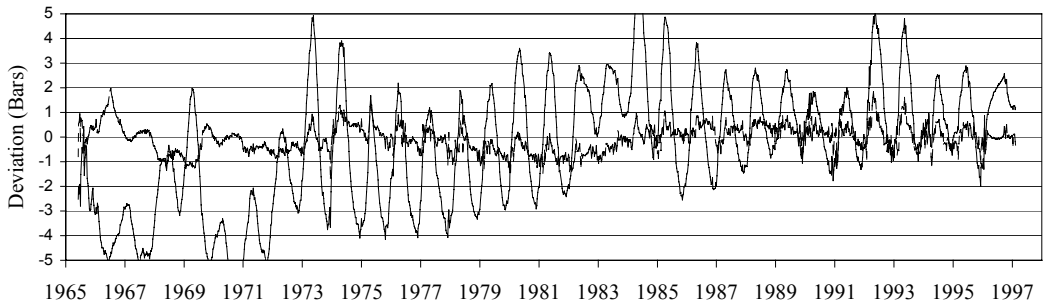

Fig. **3** Initial (large amplitude) and final (small amplitude) series of pressure deviations.

SENSITIVITY ANALYSIS

The sensitivity analysis is based on a local linearisation of the model in the vicinity of the optimal set of parameters. The method, which uses also the Hessian matrix, has been described among others by Thiéry (1993b, c), Leijnse, A. (1982). Classically the method is applied assuming that the observations, or the simulations residuals, are independent which is not acceptable most of the time. Instead Thiéry (1993b, c) shows that it is necessary to integrate their spatial or temporal dependence. This could be done using the covariance matrix of observations. However this not feasible considering the large number of observations which would yield a huge matrix. An approximate method has been used instead. The analysis of the monitored pressures shows that two pressures separated by 10 time lags (approx. 70 days) have an autocorrelation coefficient not less than 0.5. It can then be roughly considered that a set of 10 consecutive observations forms only one independent observation. The total number of independent observations reduces from 1656 to 166, which is realistic because it corresponds to 5.2 independent observations per year during 31.8 years. Using this method the standard deviation of the parameters has been calculated. Three parameters, k, ω and β_{por} had a logarithmic transformation. For these parameters relative standard deviation is displayed. The relative standard deviation is based on the standard deviation of the logarithm. It defines the range which has a probability of 68.2% to contain the true value. Table 2 displays the correlation matrix of the parameters and the standard deviations. It appears that two parameters are precisely defined : k and ω (std deviation 3 to 5%). Two parameters are

strongly correlated k and β_{por} (correlation –0.923). The negative sign indicates that a large compressibility has a similar effect as a large permeability : both parameters tend to dampen pressure variations.

Table 2 Sensitivity analysis of the parameters.

	Permeability	Porosity	Head Correction	Compressi-bility	Optimal Value	Standard Deviation	Relative Standard Deviation
Permeability	1				0.729		4.89 %
Porosity	0.064	1			22.45		2.74 %
Head correction	0.619	0.791	1		-7.89	2.55 m	
Compressibility	-0.923*	0.054	-0.462	1	0.857		34.5 %

The sensitivity analysis shows that the rock compressibility is not very well identified. This parameter could be fixed to a reference value. The other parameters would then be still better identified.

CONCLUSIONS

The multiphase flow numerical method used in MARTHE code performs well and the automatic calibration scheme is efficient even in highly non linear systems. It enables to simulate very accurately the variations of gas pressure in a storage during a long monitored period which proves its ability to yield reliable predictions. The presented sensitivity analysis is useful in determining the most significant parameters.

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