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## SALTWATER INTRUSION MODELLING WITH AN EFFICIENT MULTIPHASE APPROACH: THEORY AND SEVERAL FIELD APPLICATIONS

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### Abstract

Groundwater resources in Mediterranean coasts are often limited by saltwater intrusion. In order to assess accurately the available resources it is usually necessary to model the consequences of implementing new wells in the aquifer. A common approach for taking into account the presence of saltwater is to use a variable density model. This classical method however is plagued by several drawbacks: first the system must be modelled in 3D in order to account for the density effects, even when the aquifer could be considered as monolayer; second this scheme usually introduces artificial smearing due to numerical dispersion. An alternative very efficient method is to use a special multiphase approach with two separated fluids, freshwater and saltwater coupled with an equivalent retention law, which respects the saltwater and freshwater equilibrium in each model cell. This method, which may be used in steady state as well as in transient state has several decisive advantages: it may be used for monolayer aquifers, or if needed for multilayer systems, the interface between the two fluids is sharp without any smearing, and the calculations are very fast (often on the order of magnitude of 30 to 50 times faster than with the classical variable density method). The method has been verified with a set of available analytical solutions. It has been applied with success to field cases. Several cases are presented: in the island of Malta and in Camargue (South of France).

**Keywords:** Sea water intrusion, groundwater model, multiphase flow, sharp interface, validation, field cases.

### Introduction

Saltwater intrusion in aquifers has been studied for more than a century by a number of authors and many exact or analytical solutions were developed mainly before the conception of the numerical models (see

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Herzberg, 1901; Glover, 1959; Keulegan, 1954; Henry, 1962 and 1964; Hantush, 1968; Bear, 1972 and 1979; Strack, 1976, among others). When numerical models became available around 1970, simplified approaches with two distinct fluids, freshwater and saltwater, were derived having in mind the common sharpness of the fluids interface. More recently, models solving the coupled variable-density groundwater flow and solute-transport equation were developed and used to study saltwater encroachment. For instance MOCDENS3D (Oude Essink, 1998), SEAWAT (Guo and Langevin, 2002), SUTRA (Voss and Souza, 1987), MARTHE (Thiery, 1990, 1993) FEFLOW (Diersh 1998), and others. This single-phase approach is appropriate when there is a large vertical transition zone due to mixing by dispersion. However it has some drawbacks:

- It is generally necessary to adopt a 3D or multilayer scheme to model the density field, even when the aquifer is monolayer with quasi horizontal flows, which implies a large number of cells and long execution time
- Due to numerical dispersion, and the usually smaller vertical extension of the layers compared to the horizontal direction, the computed mixing very often tends to be too large, especially for shallow aquifers.
- The coupled variable-density flow and solute-transport is prone to oscillations and not always adapted to direct steady state calculations, for instance when using Total Variation Diminishing (TVD) methods for transport.

Actual two-phase flow models were also developed for instance: INTRANS, a monolayer model (Bonnet et al., 1974), SHARP, a quasi-three-dimensional model (Essaid, 1990), MARTHE, a full 3D model both multiphase flow or density driven model (Thiery, 1990; 1993). This approach, which applies only when the width of the freshwater-saltwater transition zone is small relative to the thickness of the aquifer, which is very often the case, has many very important attractive features:

- Possibility to adopt a monolayer scheme when physically acceptable.
- No numerical dispersion.
- Very short execution time and direct steady state calculation possible.

This paper describes the principles of the calculations and shows how a general multiphase flow model, oil-water for instance, may be transformed into a stratified flow model to simulate fresh- and saltwater flows. Then, the verification of the method with the MARTHE model is presented by comparison with analytical solutions. Finally, applications to real field cases are presented.

## Description of the numerical method

The MARTHE model is a versatile, full three-dimensional finite volume model. It is possible to choose between a classical single phase coupled variable-density flow and solute-transport scheme, or a full 3D two phase approach integrating seepage faces when necessary. It is possible to combine a full 3D approach in some layers and a quasi-3D approach in others by introducing semi-pervious aquitard layers between some aquifer layers.

### Governing equations

In each model cell there are two governing equations: one for freshwater and one for saltwater. Each equation, which states the conservation of mass of water (fresh or salt), is very similar to the standard equation of groundwater flow in non-saturated medium. These two equations are coupled by the hydrostatic relation which determines the position of the interface and hence the saturation of each phase. Each fluid being of constant density, the exchanges of mass are proportional to the volumetric flows. The following equations are written for a finite difference scheme:

Freshwater equation: three dimensional conservation of freshwater mass: (subscript f)

$$\sum_i^6 T_{fi} \cdot (H_{fi} - H_{fc}) + Q_f = \frac{d(STO_f)}{dt}$$

Saltwater equation: three dimensional conservation of saltwater mass: (subscript s)

$$\sum_i^6 T_{si} \cdot (H_{si} - H_{sc}) + Q_s = \frac{d(STO_s)}{dt}$$

with:

T = exchange coefficient	[L <sup>2</sup> T <sup>-1</sup> ]	H = "true" hydraulic head	[L]
Q = inflow of fluid	[L <sup>3</sup> T <sup>-1</sup> ]	STO = stored volume of fluid	[L <sup>3</sup> ]
t = Time	[T]		

Indices: f = freshwater; s = saltwater  
 c = centre cell; i = index of one of the 6 cells adjacent to the centre cell in 3D  
 p = previous time step (centre cell)

$T_{fi}$	= $(K_{fi} \cdot A_i / dx_i)$ averaged between cell i and cell c
$T_{si}$	= $(K_{si} \cdot A_i / dx_i)$ averaged between cell i and cell c
$K_{fi}$	= $k \cdot (\rho_w \cdot g / \mu_f) \cdot \theta_{fi}$ ; $K_{si} = k \cdot (\rho_w \cdot g / \mu_s) \cdot \theta_{si}$
$STO_f$	= $Vol \cdot [(\theta_f - \theta_{fp})] + S_s \cdot (H_{fc} - H_{fcp})$
$STO_s$	= $Vol \cdot [(\theta_s - \theta_{sp})] + S_s \cdot d_s \cdot (H_{sc} - H_{scp})$
$H_f, H_s$	= freshwater and saltwater "true" hydraulic head at the centre of the cell [L]
$H_f$	= $h_f + z$ ; $H_s = h_s / d_s + z$
$h_f, h_s$	= pressure head at the centre of the cell expressed in freshwater height [L]

with:

A = exchange area between cells	[L <sup>2</sup> ]	x = distance	[L]
z = altitude of the model cell centre	[L]	dz = vertical size of the cell	[L]
Vol = volume of the cell	[L <sup>3</sup> ]	$S_s$ = specific storage coefficient	[L <sup>-1</sup> ]
g = gravity acceleration	[LT <sup>-2</sup> ]	k = permeability	[L <sup>2</sup> ]
$\rho$ = fluid density	[ML <sup>-3</sup> ]	$\mu$ = viscosity	[ML <sup>-1</sup> T <sup>-1</sup> ]

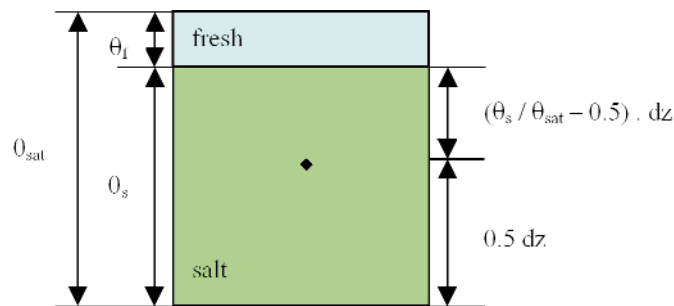
$$d_s = \text{saltwater relative density} = \rho_s / \rho_f [-] \quad \rho_w = \text{water density in standard conditions of pressure and temperature} \quad [\text{ML}^{-3}]$$

$$\theta_f = \text{freshwater volumetric content in a cell} [-] \quad \theta_s = \text{saltwater volumetric content in a cell} [-]$$

### Equations coupling

Using a Finite Difference scheme, it is necessary to compute the pressure heads  $h_f$  and  $h_s$  in the centre of the cell, at an elevation  $dz/2$  above the cell floor,  $dz$  being the height of the cell. In each cell, when freshwater and saltwater both are present, freshwater is situated above saltwater and the pressure distribution is hydrostatic within the cell. This gives the following relations:

- *Confined cells:*  $\theta_f + \theta_s = \theta_{\text{sat}}$  = porosity. See Figure 1.



**Figure 1.** Phases relations in a confined cell.

The interface elevation above the floor of the cell is  $z_i = dz \cdot \theta_s / \theta_{\text{sat}}$ . At this location the pressure heads  $h_i$  are equal in both phases. At mid elevation the hydrostatic relation yields:

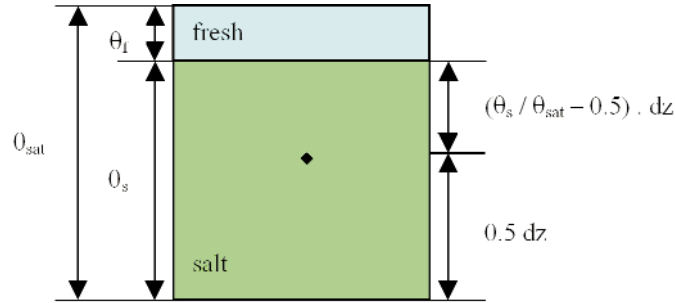
- Freshwater:  $h_f = h_i - (dz/2 - z_i) \cdot 1$
- Saltwater:  $h_s = h_i - (dz/2 - z_i) \cdot d_s$

$$h_s - h_f = (\theta_s / \theta_{\text{sat}} - 0.5) \cdot (d_s - 1) \cdot dz$$

Knowing  $h_s$  and  $h_f$ , the relation yields  $\theta_s$  hence  $\theta_f = \theta_{\text{sat}} - \theta_s$ .

- When  $(h_s - h_f) \geq 0.5 \cdot (d_s - 1) \cdot dz \Rightarrow \theta_s / \theta_{\text{sat}} = 1.0 \Rightarrow$  the freshwater phase disappears
- When  $(h_f - h_s) \geq 0.5 \cdot (d_s - 1) \cdot dz \Rightarrow \theta_s / \theta_{\text{sat}} = 0.0 \Rightarrow$  the saltwater phase disappears

- *Unconfined cells:*  $\theta_f + \theta_s = \theta_{liq} < \theta_{sat}$ . See Figure 2.



**Figure 2.** Phases relations in an unconfined cell.

There are three phases: air + freshwater + saltwater, but assuming that air is at atmospheric pressure only two phases are considered. The air interface elevation is  $z_a = dz \cdot (\theta_s + \theta_f) / \theta_{sat}$ . At this location freshwater is in contact with air and the pressure head is equal to zero which enables to compute the freshwater pressure head at mid elevation  $dz/2$ :

$$h_f = 0 - (dz/2 - z_a) \cdot \gamma = [(\theta_f + \theta_s) / \theta_{sat} - 0.5] \cdot dz \quad [1]$$

As for confined cells, the hydrostatic relation between freshwater and saltwater yields the same equation:

$$h_s - h_f = (\theta_s / \theta_{sat} - 0.5) \cdot (d_s - 1) \cdot dz \quad [2]$$

Knowing  $h_s$  and  $h_f$ , the first relation yields  $(\theta_f + \theta_s)$ , and the second one yields  $\theta_f$ . Hence in both phases, the water contents  $\theta_f$  and  $\theta_s$  are known.

- When  $h_f \leq -0.5 \cdot dz \Rightarrow (\theta_f + \theta_s) = 0.0 \Rightarrow$  no more fluid: both freshwater and saltwater phases disappear.
- When  $h_f \geq 0.5 \cdot dz \Rightarrow (\theta_f + \theta_s) = \theta_{sat} \Rightarrow$  the cell becomes confined

In both cases, confined or unconfined, letting  $h_{capil} = h_s - h_f$ , one gets a relation in the form:

$h_{capil} = f(\theta_f)$  which is equivalent to a retention law in a classical multiphase flow approach. There are however two important differences from the classical multiphase formulations:

- Instead of being always positive, the capillary pressure (or «suction») is positive or negative depending on the freshwater saturation  $\theta_f$ . This implies that when the freshwater relative saturation is less than 50 %, saltwater has a higher pressure head and plays the role of the Non Aqueous Phase (NAQ). On the other hand, when the freshwater relative saturation is greater than 50 %, it is freshwater which has a higher pressure head and plays the role of the NAQ phase.
- The parameter of the "equivalent" retention law does not depend only on the porous medium and on the fluids characteristics. It also depends on the vertical dimension of the model cell.

### ***Solution of the coupled system of equations***

The equations are solved with the 3D finite volume code MARTHE described by Thiéry (1990, 1993, 2000). Three formulations are available for the unknowns: "freshwater head – freshwater content" ( $H_f, \theta_f$ ) or "saltwater head – freshwater content" ( $H_s, \theta_f$ ) or "freshwater head – saltwater head" ( $H_f, H_s$ ). In steady state however, only the "freshwater head – saltwater head" formulation is possible. Picard iterations are used to take care of the non-linearities. The transfer coefficient between two adjacent model cells uses an upstream weighting of the adjacent water contents (i.e. it is the water content of the upstream cell which is used). This scheme has proved to be very stable and efficient. The default formulation is the ( $H_f, H_s$ ) formulation, which simultaneously solves the two systems of equations which has proved most of the time to be the best.

### ***Phase disappearance***

As outlined above, according to the capillary pressure one phase (or even both phases) may disappear. For instance in the upstream region, or upper region, there is often only freshwater. On the other hand, in the downstream, or lower region, there is often only saltwater. This is taken into account classically in multiphase flow models and causes no problems. One method is to consider that there is always a very small minimum relative saturation in each phase, on the order of magnitude of  $10^{-5}$ , in order to insure hydraulic continuity and flow in the "disappeared" phase and to permit a future re-saturation. Another method is to consider that there is always a very small minimal equivalent exchange coefficient.

### **Verification**

The presented multiphase approach has been verified on a number of analytical solutions in order to verify and outline its applicability. Two very classical saltwater intrusion problems, namely Henry test (Henry, 1962, 1964) without diffusion and the steady state "Glover wedge" (Glover, 1959) could be simulated without problems and are not presented here. Some verification examples are presented below in 1D, 2D or 3D, confined or unconfined aquifer, steady state or transient state.

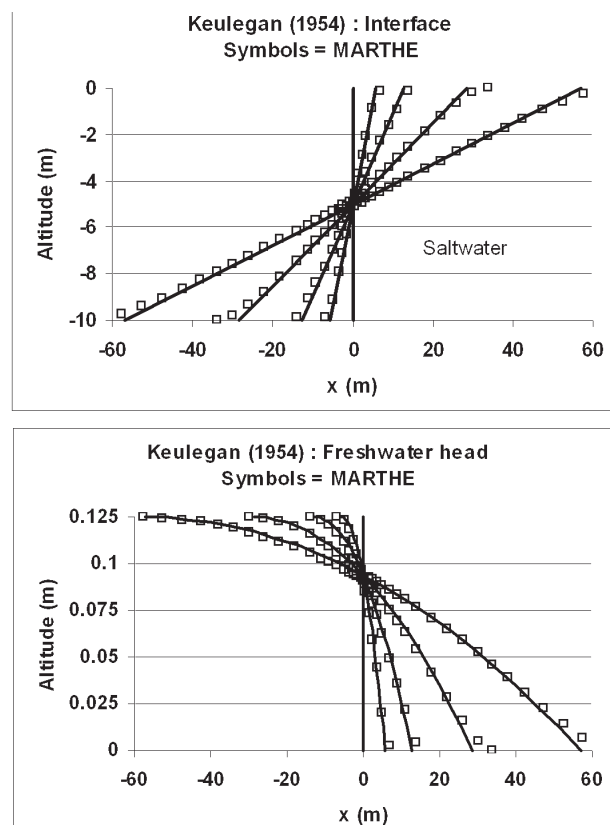
### ***Confined aquifer, transient state: Keulegan test***

This test described by Keulegan (1954), who gives an analytical solution, is a difficult one that has been studied by Shamir and Dagan (1971), and by Mercer et al. (1981). It is relative to a confined aquifer of uniform thickness and infinite lateral extension. Initially there is a vertical interface, maintained by a gate, separating two compartments with freshwater in the left one and saltwater in the right one. At  $t = 0$  the gate is removed and the interface begins to move owing to the density difference. The analytical solution neglects vertical head losses and the scheme is 1D. The test is a difficult one because there is a shock at initial time and the movement is fast. The following parameters have been selected: aquifer thickness = 10 m, hydraulic conductivity = 39.8 m/day, porosity = 30 %. The test has been modelled in transient state with adaptable time steps from 0.05 to 10 days. The 1D grid which is 140 m in length is composed of 44 cells of variable width, mainly from 2 to 4 m. Figure 3 shows that the simulated position of the interface and freshwater head are quite close to the analytical solution. It has been verified that the Ghyben-

Herzberg (Herzberg, 1901) approximation doesn't apply to all, which is obvious from the graph: the interface variation amplitude is 10 m which is double than the freshwater head amplitude multiplied by  $40 = 1 / (ds - 1)$ .

### **Unconfined aquifer, steady state: Strack test**

This test is described by Strack (1976) who gives an analytical solution. The test relates to a shallow coastal unconfined aquifer in direct connection to the sea. There is an upstream flow of freshwater towards the sea and a single pumping well is situated in the freshwater body. Figure 4 illustrates this test: the bottom of the aquifer is 20 m below the sea level, the pumping well is situated 600 m from the coast and is screened over the whole thickness. The analytical solution neglects vertical head losses and the scheme is 2D. The following parameters have been selected for this test: hydraulic conductivity = 70 m/day, upstream flow = 1 m<sup>3</sup>/s per unit of width, pumping rate = 400 m<sup>3</sup>/s, porosity = 10%. The test has been modelled in steady state with a plane 2D grid comprising 47 rows of 63 columns of variable size. Due to the symmetry only one half of the domain was modelled with an extension of 2000 m in each direction. Figure 5 shows that the simulation of this case with 3 phases is quite accurate. The effect of the pumping well can be readily observed.



**Figure 3.** Keulegan test: comparison with the analytical solution. Time variations  $t = 1, 5, 25$  and 100 days. Top = interface; Bottom = freshwater head.



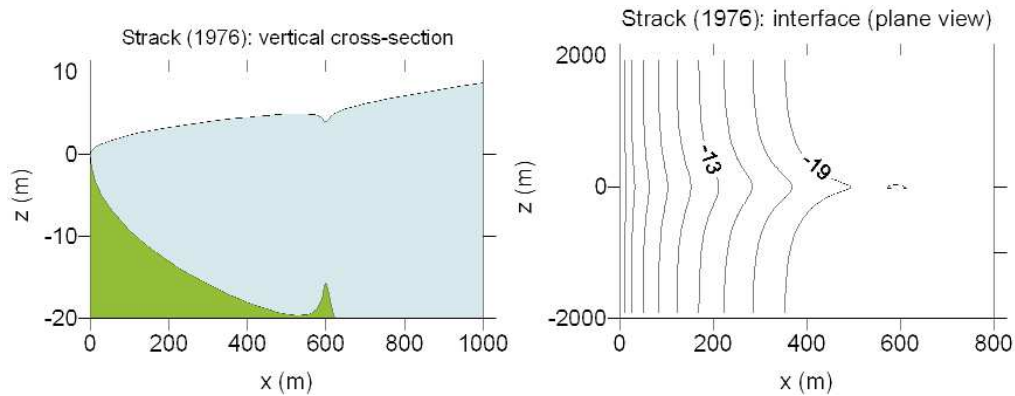


Figure 4. Strack test description.

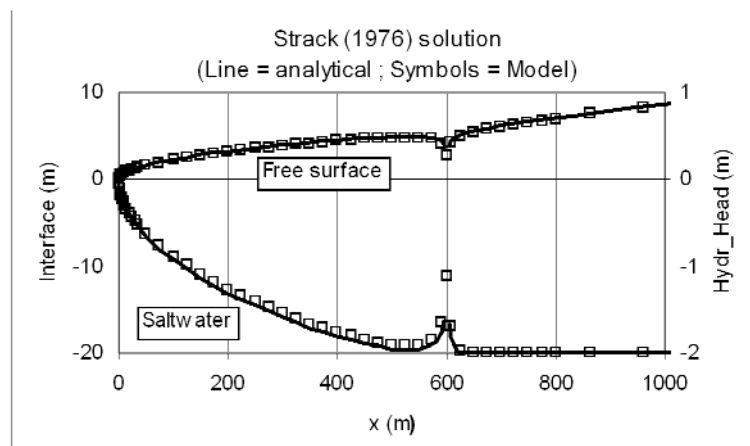


Figure 5. Comparison with the analytical solution of Strack (1976).

### ***Unconfined aquifer, transient state: Hantush test***

This test is described by Hantush (1968) who gives an approximate analytical solution. The test relates to unsteady intrusion into a thick unconfined coastal aquifer. Initially the aquifer is saturated with saltwater. At time  $t = 0$ , a recharge is applied over a strip parallel to the sea. As shown on Figure 6, a freshwater lens begins to develop, reaches the seashore after some time and then extends landwards in the opposite direction before reaching a steady state. The analytical solution neglects vertical head losses in the saturated region and the scheme is 1D. The following parameters have been selected for this test: aquifer thickness = 50 m, lateral width = 100 m, recharge strip 10 m wide situated 45 m from the coast, recharge rate 0.02 m/d, porosity = 10%, hydraulic conductivity = 100 m/d. The evolution of the interface position has been simulated during 5000 days. The test has been modelled in transient state with adaptable time steps. The 1D grid, which is 100 m in length, is composed of 102 cells of 1 m width except for two finer cells near the sea limit. Figure 7 compares the simulated interface after 10, 25, 50, 100, 200 and 5000 days with the approximate solution. This figure shows a correct simulation of this unconfined transient test.

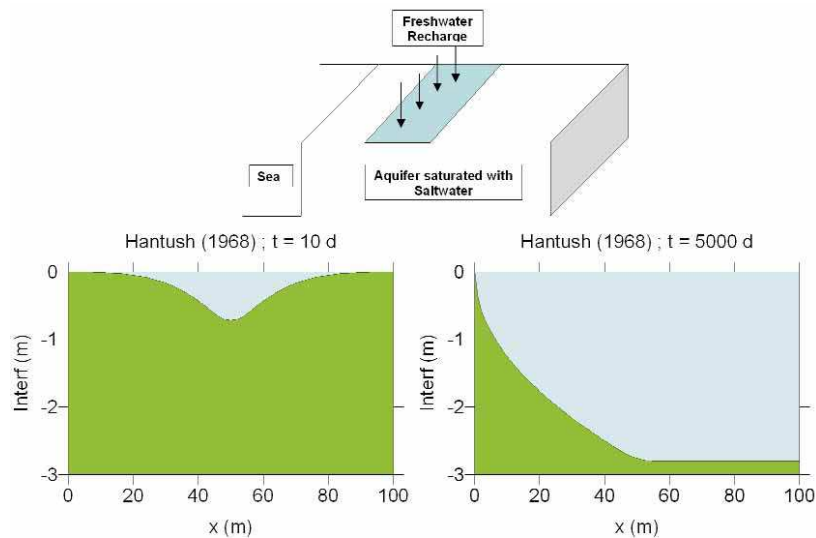


Figure 6. Hantush test: top: system description. Bottom: interface position at two dates.

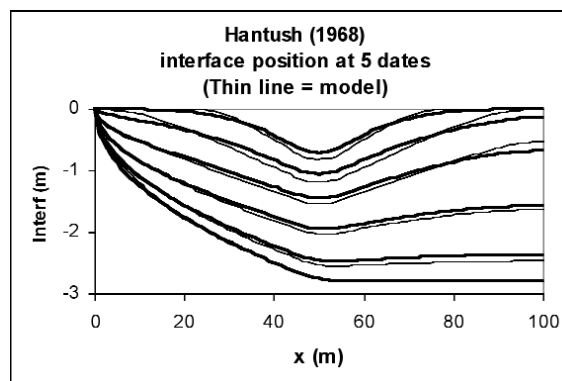
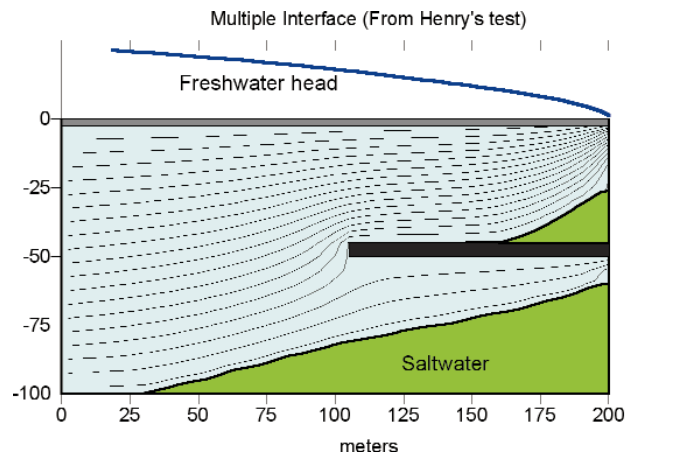


Figure 7. Hantush test: comparison of the simulated interface after 10, 25, 50, 100, 200 and 5000 days with the approximate analytical solution.

### 3D Steady state Henry test with a seepage face and several interfaces

This test, which has no known analytical solution, has been inspired from Henry (1964) test. It was designed to illustrate the possibility of having complicated vertical geometry with a seepage face and several interfaces. The following parameters have been selected for this test: confined aquifer of uniform thickness = 100 m and lateral extension = 200 m, hydraulic conductivity = 0.01 m/s, porosity = 35%. There is an upstream flow =  $6.6 \cdot 10^{-3} \text{ m}^3/\text{s}$  per unit width at the left limit (Darcy velocity =  $6.6 \cdot 10^{-5} \text{ m/s}$ ), a vertical seepage face at the right limit and an impermeable horizontal barrier extending from  $x = 105 \text{ m}$  to  $200 \text{ m}$  at a depth between 45 m and 50 m. The modelling, which takes explicitly into account the vertical velocities, is 3D; however only a 2D cross section is modelled. The test has been modelled in steady

state. The grid is composed of 20 layers of 5 m thickness and 41 columns of 5 m width, except for one cell 0.1 m wide at the sea limit. Figure 8 describes the test and displays the two computed salt wedges.



**Figure 8.** Theoretical test with multiple interfaces and a seepage face.

### ***Conclusion of the verifications tests***

The verification tests proved the efficiency of the described approach. It is possible to model flows in 3D, taking into account the vertical velocity components, in confined or unconfined formations, in steady state or in transient state. Seepage faces may be incorporated. Pumping wells or injection wells may be introduced in the freshwater phase or in the saltwater or in both. The calculations are very fast and it is possible to model simple aquifers as a monolayer system when the flows are quasi-horizontal.

### **Field case**

#### ***The Malta island aquifer system***

This is the Malta island aquifer that was studied and modelled by Gutierrez (1994, 1996). The aquifer system was first modelled with a variable-density scheme in 3D with 15 layers and a total of 36 000 square cells of variable thickness during 24 years from 1966 to 1990. Later, a two-phase approach was used with a monolayer geometry. The aquifer system was modelled in transient state with adaptable time steps from 2 to 25 days. The 2D grid is composed of 48 rows and 50 columns of square cells of 500 m size. Figure 9 shows the island with the pumping system by deep galleries (appearing as stars on the figure) and the position of a vertical cross-section through a pumping zone. The figure displays the interface depth calculated with the two-phase approach. It shows (pale grey colour) that the interface is very high near the pumping galleries. Figure 10 displays the vertical cross-section of the heads and the time variations of the saltwater interface. Obviously the interface rose significantly between 1966 and 1990 due to the increase in the pumping rate. Both freshwater and saltwater were pumped simultaneously in some

galleries, which was simulated as observed. The results obtained with the two-phase modelling are quite similar, with a number of cell 15 times smaller and a computation time divided by 50, to the results obtained with the variable density scheme considering the 50% salinity isoline.

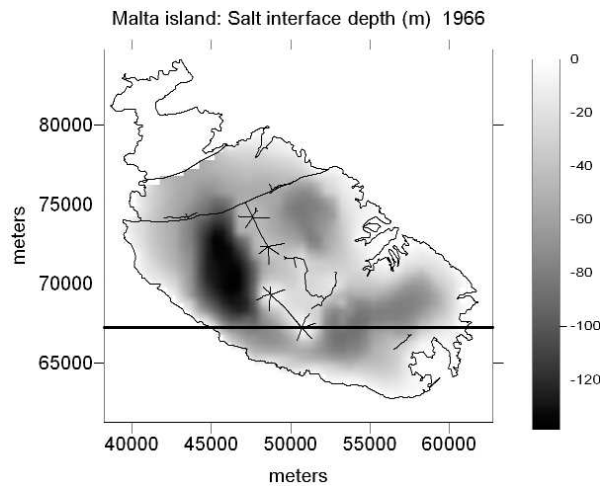


Figure 9. Malta island modelling. Saltwater interface depth (m) in 1966.

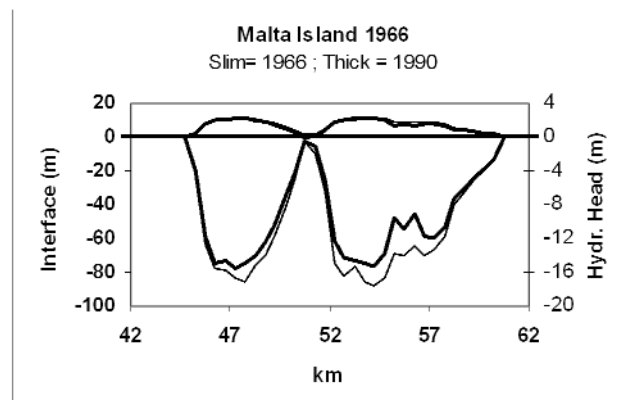
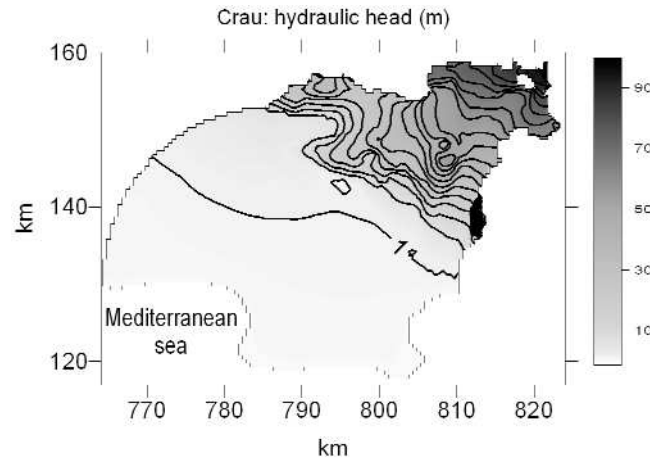


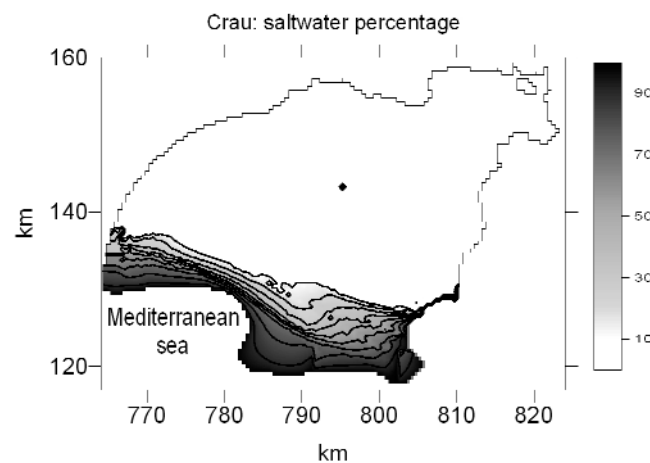
Figure 10. Malta island modelling. Vertical cross-section displaying: the hydraulic head (top) and the saltwater interface variations between 1966 and 1990.

### ***The Crau aquifer (Camargue – France)***

The southern border of this aquifer is the Mediterranean Sea. As can be seen in Figure 11, the hydraulic head near the sea is very flat: it is the Crau plain. A strip larger than 20 km has a hydraulic head between 0 and 1 m. As a consequence of this disposition, there is a large saltwater intrusion as shown on Figure 12. Even with this very flat configuration there were no difficulties in the calculations. The simulation was performed in steady state with a grid composed of 88 rows and 120 columns of square cells of 500 m size.



**Figure 11.** Crau (Camargue – France) aquifer: computed hydraulic head field.



**Figure 12.** Crau (Camargue – France) aquifer: computed percent of saltwater.

## Conclusions

It has been shown that the two-phase flow method described in the paper is efficient and accurate. Although it can be applied only when the width of the freshwater-saltwater transition zone is small, because it neglects dispersion, it displays a number of advantages. It is possible to adopt a monolayer scheme when it is physically acceptable, which dramatically reduces the number of cells. There is no numerical dispersion. The resolution scheme is stable and the computation times are short, often 30 to 50 times less than with the variable density method. It is possible to compute directly a steady state. The two-phase method presented in this paper can be also used to simulate Dense Non Aqueous Phase Liquid (DNAPL) flows in aquifers.

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