Velocity structure inversion from H/V spectral ratios of earthquake data: Application to the Tohoku region, Japan

Ariane Ducellier, Hiroshi Kawase, Shinichi Matsushima

VELOCITY STRUCTURE INVERSION FROM H/V SPECTRAL RATIOS OF EARTHQUAKE DATA: APPLICATION TO THE TOHOKU REGION, JAPAN

Ariane Ducellier, Hiroshi Kawase, Shinichi Matsushima

SUMMARY:
In this study we focus on sites where the site effect for seismic ground motionscan be described using a one dimensional model. Previous studies show that the imaginary part of the Green's function at the free surface is proportional to the square of the absolute value of the corresponding transfer function for a plane, vertically incident wave with unit amplitude. It is then possible to carry out an inversion of the 1D velocity structure using the relationship between the horizontal-to-vertical (H/V) spectral ratio and the ratio of horizontal and vertical transfer functions. We carry out inversions of the velocity structure for three sites of the KiK-net network in the Tohoku area, Japan, following the proposed theory for earthquake H/V ratios. We verify that there is a good match between the earthquake H/V ratio and the transfer function corresponding to the new proposed velocity structure for the three sites studied in the present work.

Keywords: H/V spectral ratio, site effects, inversion

1. INTRODUCTION

Multiple scattering waves sample the medium along their multiple paths. In the acoustic case, a statistical approach for wave propagation led to the concept of diffuse fields. In the seismic case, a uniformly distributed set of random forces provides an equipartitioned illumination of the medium (e.g. Sánchez-Sesma*et al.*, 2008). The seismic field engendered by such equipartitioned illumination behaves as a diffusion-like regime. When the seismic motion is dominated by multiple scattering of waves, the energy densities have therefore a diffusion-like behavior (Margerin*et al.*, 2009). Under these conditions, we can retrieve the Green's function from averaging cross-correlations of the recorded motion of such diffuse field (Sánchez-Sesma and Campillo, 2006).

In this study, we assume that the seismic field engendered by the summation of a sufficiently large number of seismic events covering a large interval of azimuths and incidence angles can be considered as a diffuse-wave field. A new formulation for the average horizontal-to-vertical (H/V) spectral ratio of earthquake motions was then constructed by Kawase*et al.* (2011). This leads to a new method to retrieve the underneath velocity structure.

We consider strong ground motion in elastic layered media. Kawase*et al.* (2011) explored the theoretical consequences of assuming both a "sufficiently" flat layered site and "sufficiently" deep earthquake sources, such that surface waves are negligible. Under these conditions, the illumination is produced by incident plane waves and a one dimensional description of the wave propagation can be applied. The ground motion is supposed to be spatially homogeneous in a statistical sense, that is to say the average of normalized ground motion spectral densities will depend only on depth. By summing a sufficiently large number of earthquake motions, the averaged autocorrelations can thus be related through the diffuse field formalism to the imaginary part of the one dimensional Green's function, when the source and the receiver are both at the same point.

Claerbout (1968) established a relationship between reflection response and autocorrelation of surface motion for one dimensional layered media. Kawase*et al.* (2011) interpreted these pioneering results under the light of diffuse-field concepts and proposed an extension of Claerbout's results. They assert that the imaginary part of the one dimensional Green's function at the surface for a surface source is equal to the square of the absolute value of the transfer function for an incoming unit displacement divided by four times the circular frequency and the half-space impedance.

By connecting this last result with the illumination due to earthquakes, Kawase*et al.* (2011) established a relationship between the average horizontal-to-vertical (H/V) spectral ratio of earthquake motions and the ratio of transfer functions for P- and S-waves. In this study, we focus on three sites from the KiK-net network in the Tohoku area, Japan. We can compute the theoretical transfer function of a soil column from the soil layers' mechanical properties using the Thomson-Haskell propagator matrix method (Thomson, 1950; Haskell, 1953). We then carry out inversions of the velocity structure for the three sites considered and verify that there is a good match between the earthquake H/V spectral ratio and the transfer function corresponding to the new velocity structure obtained from the inversion.

2. INVERSION METHOD

From the relationship between the average autocorrelation of the recorded motions and the one dimensional Green's function, Kawase*et al.* (2011) deduced that the average horizontal-to-vertical (H/V) spectral ratio can be written as follows:

$$\frac{H(x,\omega)}{V(x,\omega)} = \sqrt{\frac{Im\left(G_{11}^{1D}(x,x,\omega)\right)+Im\left(G_{22}^{1D}(x,x,\omega)\right)}{Im\left(G_{33}^{1D}(x,x,\omega)\right)}} \tag{2.1}$$

where$\frac{H(x,\omega)}{V(x,\omega)}$ is the average H/V spectral ratio at a receiver *x* for frequency ω and $G_{ii}^{1D}(x,x,\omega)$ is the i^{th} component of the one dimensional Green's function at a source *x* and a receiver *x* for frequency ω. As in a one dimensional description of the wave propagation both horizontal components of the Green's function are equal, Equation (2.1) can be simplified as:

$$\frac{H(x,\omega)}{V(x,\omega)} = \sqrt{\frac{2Im\left(G_{11}^{1D}(x,x,\omega)\right)}{Im\left(G_{33}^{1D}(x,x,\omega)\right)}} \tag{2.2}$$

The relationship established by Kawase*et al.* (2011) between the imaginary part of the one dimensional Green's function and the transfer function for an incoming unit displacement can be written as:

$$Im[G_{ii}^{1D}(x,x,\omega)] = (4\omega\rho_H c_H)^{-1}|TF_i(x,\omega)|^2 \tag{2.3}$$

where $TF_i(x,\omega)$is the i^{th} component of the transfer function between the top of the half-space and the position *x*, ρ_H is the density of the half-space and c_H is eitherthe P- or S-wave velocity depending on whether we consider a vertical or an horizontal displacement.

By specializing Equation (2.3) for both horizontal and vertical motion, we can express the H/V spectral ratio at the soil surface (x = 0) and on top of the half-space ($x = h$) with the transfer function for the horizontal motion due to the S-wave $TF_1(\omega)$, and for the vertical motion due to the P-wave $TF_3(\omega)$. We get:

$$\frac{H(0,\omega)}{V(0,\omega)} = \sqrt{\frac{2\alpha_H}{\beta_H}}\frac{|TF_1(0,\omega)|}{|TF_3(0,\omega)|} \text{and} \frac{H(h,\omega)}{V(h,\omega)} = \sqrt{\frac{2\alpha_H}{\beta_H}}\frac{|TF_1(h,\omega)|}{|TF_3(h,\omega)|} \tag{2.4}$$

whereα_H is the P-wave velocity and β_H is the S-wave velocityof the half-space (bedrock).

The Thomson-Haskell propagator matrix method is a frequency domain method due to Thomson (1950) and corrected by Haskell (1953). A generalization of the method was introduced in seismology by Gilbert and Backus (1966). The Thomson-Haskell method uses the equations of motion to establish the relationship between the motion-stress vector at depth z_1 and the one at depth z_2. As long as we know the mechanical properties of the soil column (e.g. S- and P-wave velocities, density and thickness of each layer in the elastic case), we can compute easily both the horizontal and vertical transfer functions from the Thomson-Haskell method. By minimizing the difference between the ratio of transfer functions and the average H/V spectral ratio at the soil surface and on top of the half-space, it becomes possible to inverse the velocity structure below the station from the bedrock to the surface.

Genetic Algorithms use techniques inspired by evolutionary biology to find exact or approximate solutions to optimization problems. In this study, we use the Genetic Algorithm code developed in De Martin *et al.* (2010) and adapt it for the case where the H/V spectral ratio is the objective function.

3. INVERSION OF VELOCITY STRUCTURE FOR THREE STATIONS

For the three stations FKSH06, FKSH09 and IWTH19 of the KiK-net network, we invert the S-wave velocities, the P-wave velocities and the thicknesses of the soil layers. As the H/V spectral ratio depends also on the ratio of the P- and S-wave velocities of the half-space, we also invert the S-wave velocity of the half-space while the P-wave velocity is kept constant, for simplification. However, it should be noted that any couple of P- and S-wave velocities of the half-space for which the ratio between both velocities is the same, could be a valid solution.

The inversion code we use in this study is based on a Genetic Algorithm; such that the solution of the optimization problem is searched into a finite number of different possible soil columns. Each parameter of the soil column can take a finite number of discrete values.In most cases, the invertedparameter can take 32 different values, ranging from 0.25 to 1.80

Table 3.1. Inversion parameters for station FKSH06

Parameter	Layer	Initial value	Interval of variations	Rangeof values obtained from the inversion	Best value obtained from the inversion
V_S (m/s)	1	200	0.25 – 1.80 * Initial value	47.5 – 52.5	50
	2	300	0.25 – 1.80 * Initial value	283.5 – 729	405
	3	650	0.25 – 1.80 * Initial value	250.25 – 643.5	357.5
	4	1200	0.25 – 1.80 * Initial value	231 – 630	420
	5	1400	0.25 – 1.80 * Initial value	1330 – 1470	1400

	6	1700	0.50 – 1.25 * Initial value	1224 – 1564	1360
	Half-space	1700	0.50 – 1.25 * Initial value	1661.75 – 2346	1955
V_P (m/s)	1	400	0.25 – 1.80 * Initial value	252 – 1296	720
	2	600	0.25 – 1.80 * Initial value	756 – 924	840
	3	1700	0.25 – 1.80 * Initial value	1785 – 4590	2550
	4	2700	0.25 – 1.80 * Initial value	2794.5 – 3881.25	3105
	5	3200	0.25 – 1.80 * Initial value	1872 – 2288	2080
	6	3200	0.25 – 1.80 * Initial value	3944 – 6032	4640
Thickness (m)	1	1	0.25 – 1.80 * Initial value	1.5675 – 1.7325	1.65
	2	4	0.25 – 1.80 * Initial value	5.94 – 7.26	6.6
	3	7	0.25 – 1.80 * Initial value	3.465 – 13.09	7.7
	4	18	0.25 – 1.80 * Initial value	25.92 – 38.88	32.4
	5	26	0.50 – 1.25 * Initial value	28.08 – 34.32	31.2
	6	44	Not inverted		

timesthe initial value. However, forthe S-wave velocity of the last layer and the half-space, the rangeof variations is smaller, ranging from 0.50 to 1.25 the initial value, in order to avoid a negative Poisson ratio. These last parameters can take only 16 different discrete values. The initial values of the soil parameters for stations FKSH06, FKSH09 and IWTH19 are shown in Tables 1, 2 and 3 respectively (column 3). They correspond to the velocity log posted onthe KiK-net website. Tables 1, 2 and 3 show also the intervals of variations where we look for a solution for each parameter (column 4).

The thicknesses of the shallower layers are inverted but the total depth of each soil column remains constant. The thicknesses of layer 6 of FKSH09, layer 4 of FKSH09, and layer 4 of IWTH19 may then vary and are computed such that the total depth is constant.

Table 3.2. Inversion parameters for station FKSH09

Parameter	Layer	Initial value	Interval of variations	Range of values obtained from the inversion	Best value obtained from the inversion
V_S (m/s)	1	140	0.25 – 1.80 * Initial value	166.25 – 183.75	175
	2	300	0.25 – 1.80 * Initial value	324 – 396	360
	3	1930	0.25 – 1.80 * Initial value	405.3 – 926.4	579
	4	2540	0.25 – 1.80 * Initial value	1447.8 – 1600.2	1524
	5	1960	0.50 – 1.25 * Initial value	1724.8 – 2587.2	2156
	Half-space	1960	0.50 – 1.25 * Initial value	1666 – 2352	1960
V_P (m/s)	1	450	0.25 – 1.80 * Initial value	567 – 1458	810
	2	1500	0.25 – 1.80 * Initial value	1721.25 – 2632.50	2025
	3	3480	0.25 – 1.80 * Initial value	4541.4 – 5550.6	5046
	4	4950	0.25 – 1.80 * Initial value	3056.625 – 3378.375	3217.5
	5	4950	0.25 – 1.80 * Initial value	7573.5 – 11583	8910
Thickness (m)	1	2	0.25 – 1.80 * Initial value	2.945 – 3.255	3.1
	2	8	0.25 – 1.80 * Initial value	9 – 11.5	10
	3	34	0.25 – 1.80 * Initial value	47.6 – 71.4	59.5
	4	126	Not inverted		
	5	30	Not inverted		

Table 3.3. Inversion parameters for station IWTH19

Parameter	Layer	Initial value	Interval of variations	Range of values obtained from the inversion	Best value obtained from the inversion
V_S (m)	1	170	0.25 – 1.80 * Initial value	110.5 – 110.5	110.5
	2	550	0.25 – 1.80 * Initial value	235.125 – 940.5	522.5
	3	700	0.25 – 1.80 * Initial value	308 – 462	385

	4	770	0.25 – 1.80 * Initial value	215.6 – 462	308
	5	990	0.25 – 1.80 * Initial value	668.25 – 816.75	742.5
	6	1270	0.50 – 1.25 * Initial value	857.25 – 1095.375	952.5
	Half-space	1270	0.50 – 1.25 * Initial value	701.675 – 990.6	825.5
V_P (m)	1	540	0.25 – 1.80 * Initial value	573.75 – 843.75	675
	2	2240	0.25 – 1.80 * Initial value	980 – 7056	3920
	3	2240	0.25 – 1.80 * Initial value	1960 –7056	3920
	4	2240	0.25 – 1.80 * Initial value	2128 – 2352	2240
	5	2430	0.25 – 1.80 * Initial value	1500.525 – 1737.45	1579.5
	6	2920	0.25 – 1.80 * Initial value	3350.7 – 4927.5	3942
Thickness (m)	1	4	0.25 – 1.80 * Initial value	4.6 – 4.6	4.6
	2	4	0.25 – 1.80 * Initial value	0.25 – 1.8	1
	3	24	0.25 – 1.80 * Initial value	3.96 – 11.16	7.2
	4	32	Not inverted		
	5	20	Not inverted		
	6	17	Not inverted		

To evaluate if the ratio of transfer functions corresponding to a given set of soil column parameters fits well the objective H/V spectral ratio, we use the following residual:

$$Res = \frac{1}{2}\left(\frac{\int\left(HVT(\omega)-HVT_0(\omega)\right)^2 d\omega}{\int\left(HVT_0(\omega)\right)^2 d\omega} + \frac{\int\left(HVB(\omega)-HVB_0(\omega)\right)^2 d\omega}{\int\left(HVB_0(\omega)\right)^2 d\omega}\right) \qquad (3.1)$$

where $HVT_0(\omega)$ and $HVB_0(\omega)$ are the objective H/V spectral ratios obtained from the observation data at frequency ω on the top (soil surface) and the bottom (top of the half-space) of the soil column, and $HVT(\omega)$ and $HVB(\omega)$ are the theoretical ratios of horizontal and vertical transfer functions on the top and the bottom of the soil column. The integration is done over the interval 0.1 to 25 Hz for all the inversions carried out in this study.

As the Genetic Algorithm generates random numbers, two successive inversions with the same input parameters will not necessarily give the same result. Therefore, for each station, we carry out ten independent inversions with the same input files to check whether we always obtain the same best soil column at the end of the inversion. The results of the inversion for stations FKSH06, FKSH09 and IWTH19 are shown respectively on Figure 1, 2 and 3. On the left panel, the abscissa is frequency and the ordinate is the H/V spectral ratio on top of the soil column. The dashed black line corresponds to the objective H/V spectral ratio computed from the observation data, while the plain black line is the best ratio of transfer functions obtained from the ten independent inversions. On the middle panel, the ordinate is the H/V spectral ratio on bottom of the soil column. On the right panel, the abscissa is velocity and the ordinate is the depth from the surface. The two plain black lines are the S-wave velocity and the P-wave velocity of the soil column corresponding to the best ratio of transfer functions on the leftand middlepanels. The two grey lines are the initial S- and P-wave velocities. The best values of the S- and P-wave velocities and the thicknesses of soil layers obtained from the inversion are also given in Tables 1, 2 and 3 (column 6).

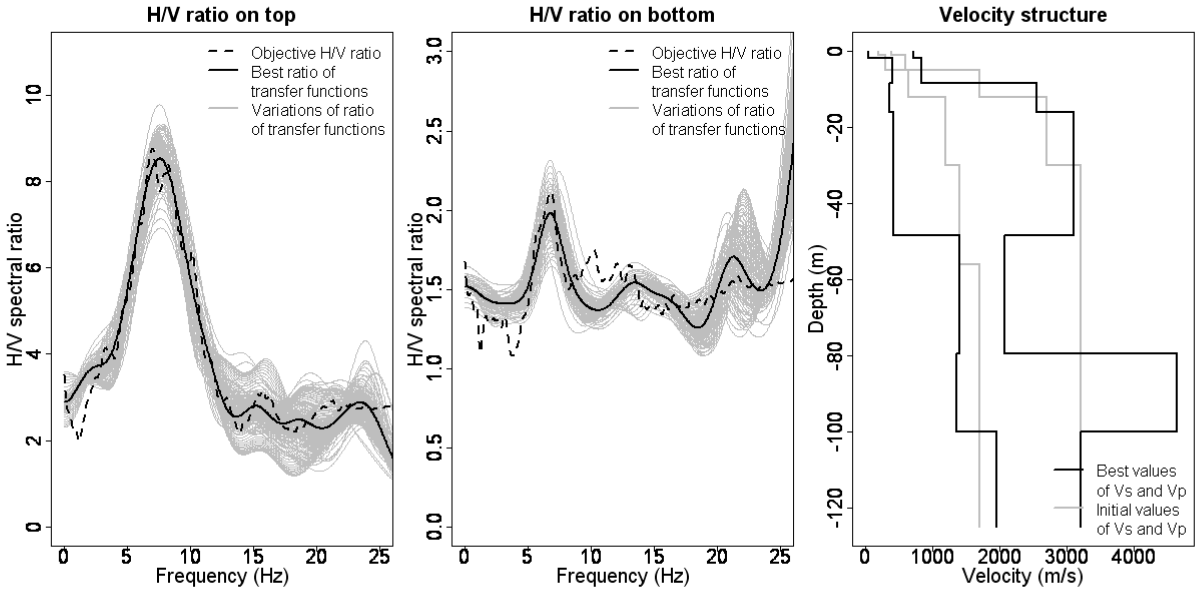

Figure 3.1. Result of inversion for station FKSH06

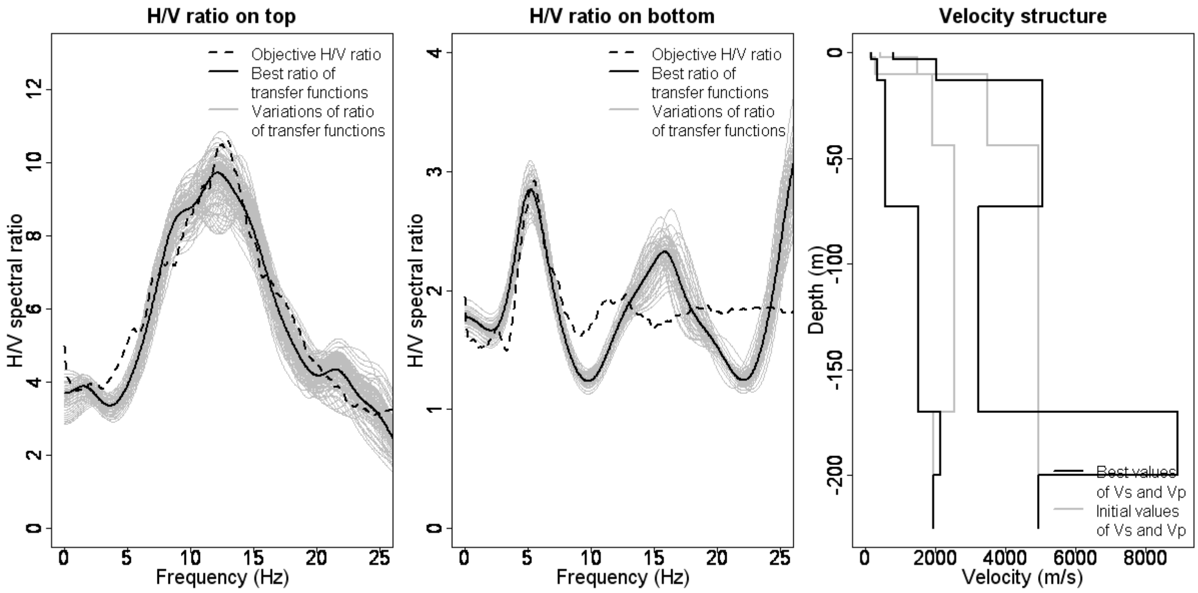

Figure 3.2. Result of inversion for station FKSH09

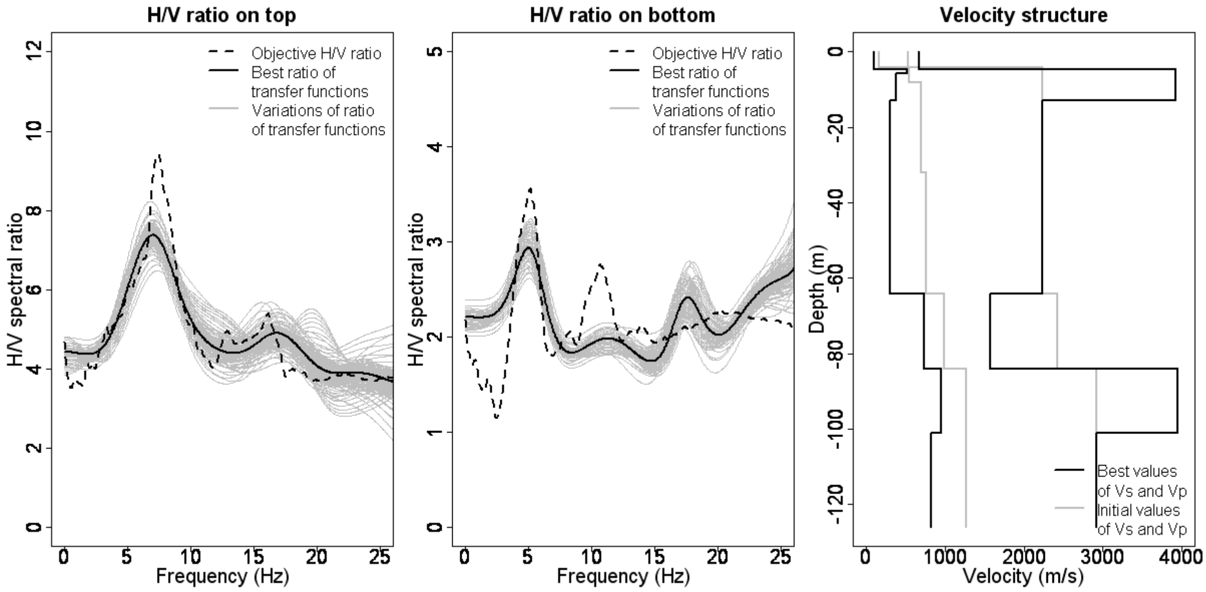

Figure 3.3. Result of inversion for station IWTH19

We have now obtained the soil column parameters corresponding to the transfer function fitting best the two H/V spectral ratios. However, all the parameters are not well constrained by the inversion process. It is possible to find soil columns whoseparameters take different values, yetcorresponding transfer function still stays very close to the best solution from the inversion. We thus take the best soil column obtained from the inversion and we make each parameter vary, one at a time, from 0.25 to 1.80 the best value, as we did during the inversion process. We then compute the corresponding ratio of transfer functions and compare it to the ratio of the best soil column.

We need to define a criterion to establish whether the ratio of a soil column is close enough to the best ratio. We use a bootstrap method to estimate the variance of the H/V spectral ratio. We select randomly NE accelerograms from the data set and compute the corresponding H/V spectral ratio. This step is repeated NB times. We then compute the mean and the variance of the NB H/V ratios thus obtained, for each frequency. In this study, we use NE = NT, NT being the total number of seismic events available in the data set. For all computations we take NB = 500. We then compute the residual between the mean H/V spectral ratio plus the square of the variance and the mean H/V spectral ratio, and take it as a target residual to compare soil columns. If the residual between the ratio of transfer functions of a given soil column and the ratio of the best soil column is lower than this target residual, we consider that both soil columns are as good a solution of the inversion problem. We check if each soil column is a good solution for both H/V spectral ratio on top and H/V spectral ratio on bottom of the soil column. The column 5 of Tables 1, 2 and 3 gives the range of values where each parameter can vary while keeping a good fit between the two objective H/V spectral ratios and the two ratios of the horizontal and vertical transfer functions. The grey lines on left and middle panels of Figures 1, 2 and 3 represent the corresponding H/V ratio on top and on bottom of the soil column. However, it should be noted that these ranges of values were established while the soil parameters varied one at a time. It does not necessarily imply that any set of parameters picked up in this table will give a ratio of transfer functions fitting well the H/V spectral ratio.

4. CONCLUSION

A series of inversions of underground 1D structure from the bedrock to the surface from three observations sites in the Tohoku region, Japan, has been carried out with a Genetic Algorithm code. A good agreement could be found between the H/V spectral ratio at the soil surface obtained with the inversion and the objective ratios. However, the agreement is not as good for the H/V spectral ratio on top of the half-space, particularly for the higher frequencies. It should be noted that the soil column parameters can take only a few different discrete values in our inversion scheme. Inverting the velocity structure with a larger number of possible values in a narrower interval of variations could lead to better results.

ACKNOWLEDGEMENT

Part of this work was carried out when one of the authors (A.D.) was staying at the Disaster Prevention Research Institute of Kyoto University, at Uji, Kyoto, Japan, supported by the Carnot funding program of the French National Research Agency (ANR) under grants Carnot 2009-ac17 and Carnot 2010-ac24.

REFERENCES

Claerbout, J. F. (1968). Synthesis of a layered medium from its acoustic transmission response.*Geophysics***33**, 264-269, doi 10.1190/1.1439927.

De Martin, F., Kawase,H. and Modaressi-FarahmandRazavi, A. (2010). Nonlinear soil response of a borehole station based on one-dimensional inversion during the 2005 Fukuoka prefecture western offshore earthquake.*Bull. Seismol. Soc. Am.***100**, 151-171, doi 10.1785/0120090125.

Gilbert, F. and Backus, G. (1966).Propagator matrices in elastic wave and vibration problems.*Geophysics***31**, 326-332.

Haskell, N. A. (1953). The dispersion of surface waves in multilayered media. *Bull. Seismol. Soc. Am.***43**, 17-34.

Kawase, H., Sánchez-Sesma, F. J. and Matsushima, S. (2011). The optimal use of horizontal-to-vertical spectral ratios of earthquake motions for velocity inversions based on diffuse-field theory for plane waves. *Bull. Seismol. Soc. Am.***101**, 2001-2014, doi 10.1785/0120100263.

Margerin, L., Campillo, M., Van Tiggelin, B. A. and Hennino, R. (2009). Energy partition of seismic coda waves in layered media: theory and application to Pinyon Flats Observatory. *Geophys. J. Int.***177**, 571-585, doi 10.1111/j.1365-246X.2008.04068.x.

Sánchez-Sesma, F. J. and Campillo, M. (2006). Retrieval of the Green's function from cross correlation: the canonical elastic problem. *Bull. Seismol. Soc. Am.***96**, 1182-1191, doi 10.1785/0120050181.

Sánchez-Sesma, F. J., Pérez-Ruiz, J. A., Luzón, F., Campillo, M. and Rodríguez-Castellanos, A. (2008). Diffuse fields in dynamic elasticity. *Wave Motion***45**, 641-654, doi 10.1016/j.wavemoti.2007.07.005.

Thomson, W. (1950).Transmission of elastic waves through a stratified solid.*J. Appl. Phys.***21**, 89-93.