Identification of modal parameters of ambient excitation structures using continuous wavelet transform

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SUMMARY:
Continuous wavelet transform (CWT) has recently emerged as a promising tool for identification of modal properties such as natural frequencies, damping ratios and mode shapes through ambient excitation measurements of structures.
This paper mainly discusses the capability of CWT method to identify the modal properties accurately using a numerical application in five storeys frame and two practical applications in a five storeys reinforced concrete structure with masonry in-fill walls and a three storeys masonry structure. Natural frequencies are identified by extracting windows paralleled to the frequency axis at wavelet ridges. Damping coefficients are estimated using the wavelet-based logarithmic decrement method. Finally, a conclusion can be drawn that the continuous wavelet transform used for the output-only system identification of ambient excitation structures yields a good agreement with results of the random decrement method and the finite element models.

Keywords: measures, continuous wavelet transform, ambient vibration

1. INTRODUCTION

Currently, ambient vibration measurements are commonly used in the assessment and structural health monitoring of civil engineering structures such as buildings, bridges and towers because the ambient vibration testing is cheap and fast, no elaborate excitation equipment are required, no boundary condition simulations are required and modal properties of the whole system can be estimated using a modal extraction technique. Furthermore, the extracted modal parameters such as natural periods, mode shapes and damping ratios can also be used for verifying the design characteristics of a civil engineering structure and validating the numerical model that can be used to predict the response of a structure under an extreme loading condition.
Many modal extraction techniques found in the literature, which are developed to estimate the modal properties using only the output response, can be categorized into three main groups as time domain, frequency domain and time-frequency domain methods. The frequency domain methods such as frequency domain decomposition (FDD) (Brincker et al. (2001)) and enhanced frequency domain decomposition (EFDD) (Brincker et al. (2001)) are commonly used for the extraction of natural frequencies and mode shapes, but uncertainty in the estimation of damping ratios. In general, time domain methods such as random decrement (RD) method (Cole (1973)), Ibrahim time domain (ITD) (Ibrahim (1977)) and the eigen-system realization algorithm (ERA) (Juang and Pappa (1985)) are widely used for damping estimation. The main problem associated with many time domain methods is how to distinguish structural modes from uncorrelated modes, which are introduced to accommodate measurement noise, leakage, out-of-band modes and non-linearity, etc. This problem can cause severe error in damping estimation.
Recently, continuous wavelet transform (CWT) method is used for modal identification of civil engineering structures using only the output response as a time-frequency domain method. As the CWT method can decompose of a signal into time-frequency domain using a mother wavelet, multi degree of freedom (MDOF) systems can be handled directly. Furthermore, it can work as a band-pass

filter and hence, this method can handle very noisy measurements (Staszewski (1997)). Another advantage of the WT method is that the stationary assumption for an ambient vibration response is not required. Therefore, this method has many advantages over the other methods in identifying modal properties using ambient vibration measurements. This method has been used successfully to extract the natural frequencies and the associated mode shapes using ambient vibration measurements of a structure (Meo et al. (2006), Le and Tamure (2009)). Regarding the damping estimation, past studies by Staszewski (1997), Hans et al. (2000), Lamarqre et al. (2000) and Ta and Lardiès (2006) have highlighted that damping ratios can be estimated adequately accurate through wavelet-based logarithmic decrement for lightly damped system. However, their studies are limited to either the impulse response or free decay response of structures.
The main objective of this study is to obtain the modal properties through ambient vibration measurements of low-rise buildings using CWT method.

2. CONTINUOUS WAVELET ANALYSIS

This section introduces a brief description on the theoretical background of the wavelet transform. However, authors strongly recommend to readers to refer the key papers to understand the theoretical background of the method well. Fourier transformation, which is a linear transformation, transform a given function x(t) in the time domain into the frequency domain using a basic function e-^{jwt}. This can be represented in the following form:

$$FT(\omega) = \int_{-\alpha}^{\alpha} x(t)\, e^{-jwt}\, dt \qquad (1)$$

This transformation does not include any local time information of the function *x(t)*. Initially, Fourier transformation of the short time sliding window is used to overcome this limitation by decomposing of the function into frequency-time domain. However, this method called Short Time Fourier Transformation (STFT) suffers from time-frequency resolution limitation. Later, CWT is developed to obtain better spectral decomposition as an alternative method to STFT. The basic idea of the CWT is to find a function *ψ(t)*, which can generate a basic for the entire domain of function *x(t)*, if a function *x(t)* satisfies the condition that *x(t)* decays to zero at +-α as in the case of Fourier transform. The fast decay in time domain and the limited band width in frequency domain introduces locality into the analysis, which is not the case of Fourier transform where a global representation can only be obtained. The function ψ(t) that is called as an wavelet must satisfy the two admissibility condition:
(1) ψ(t) must be absolutely integral and square integral:

$$\int \left|\psi(t)\right| dt < \infty$$
$$\int \left|\psi(t)\right|^2 dt < \infty \qquad (2)$$

(2) ψ(t) must be band limited and has zero mean:

$$\int \left|\frac{\hat{\psi}(\omega)}{\omega}\right| d\omega < \infty \qquad (3)$$

Then, using a wavelet, CWT can be used to decompose of a function x(t) into frequency-time domain as defined in the following form:

$$W_{(a,b)} = \frac{1}{\sqrt{a}} \int_{-\alpha}^{+\alpha} x(t)\psi^*\left(\frac{t-b}{a}\right)dt \tag{4}$$

where ψ*(t) and b are the complex conjugate of ψ(t) and the parameter localizing the wavelet function in the time domain, respectively and W(a,b) are the CWT coefficients that represent the measure of the similitude between the function x(t) and the wavelet at the time b and the scale a.
The complex Morlet wavelet is commonly used for continuous wave transform as a basic function. The complex Morlet wavelet and its Fourier transform can be expressed as:

$$\psi(t) = \frac{1}{\sqrt{2\pi}} e^{2\pi i f_c t} e^{-\frac{t^2}{2}} \tag{5}$$

$$\hat{\psi}(af) = \frac{1}{\sqrt{2\pi}} e^{(2\pi^2(af - f_c))} \tag{6}$$

f and fc are Fourier frequency and central wavelet frequency. The conversion between Fourier frequency and the scale a can be established as:

$$f = \frac{f_s f_c}{a} \tag{7}$$

where fs and a are the sampling frequency and the scale, respectively.

3. NUMERICAL APPLICATION IN A FIVE STOREY FRAME

The first objective of this part of the study is to validate the capability of CWT method to estimate natural frequencies and damping ratios accurately by decomposing of a time domain free decay response of a MDOF system into a time-frequency domain.
However, if the response is not free decay, then the two steps procedure is used to estimate the damping ratio in this study. This procedure could cause severe error in damping estimation when the response is non-stationary and recorded in a relatively short duration because RD signatures evaluated in the first step of the procedure may not be proportional to the free decay response. Therefore, the second objective is to quantify the error associated in estimation of damping ratio using the two steps procedure when the response is non-stationary and recorded in a relatively short duration.
The third objective is to quantify the error associated in estimation of natural frequencies and mode shapes when the response is non-stationary and lasts in a short duration. It must be noted that frequencies are estimated using single step procedure by extracting a window parallel to the frequency axis of time-frequency plot at wavelet ridges as described in section 2.
For these purposes, a 2D numerical model of a five storey frame to represent a MDOF system is analysed linear elastically for 50 ground motions, recorded during the past earthquakes. The stiffness proportional damping modal with 5% of critical damping for the 1st mode of vibration is assumed in the numerical modal. From the modal analysis, the identified frequencies of the first five translational modes are 2.1, 8.2, 16.1, 27.8 and 38.5Hz. Figure 1 illustrates the first five translational mode shapes.

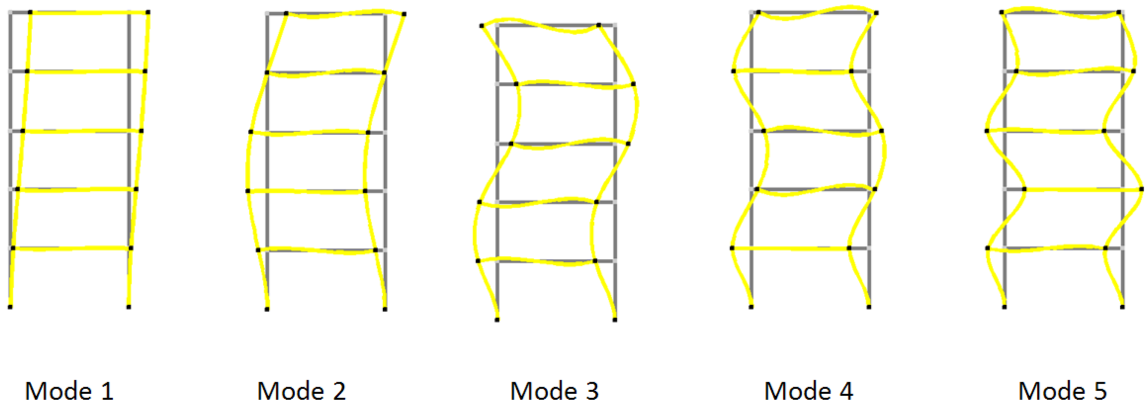

Figure 1. Mode shapes of the five translational modes

3. VALIDATING THE CAPABILITY OF CWT METHOD TO ESTIMATE THE MODAL PROPERTIES

To fulfill the first objective of validating the capability of CWT method to estimate modal properties, free decay responses are used because they avoids the effects of uncertainty in evaluation of free decay responses from other types of vibration responses (for an example, evaluation of a free decay response from an ambient vibration response of a structure using RD method). Therefore, the structure is analyzed for additional 10 seconds after each earthquake to obtain free decay responses at each storey level. Then, the free decay responses are decomposed into a time-frequency domain by using CWT with complex Morlet wavelet. In order to identify the natural frequencies, windows parallel to the frequency axis at the wavelet ridges in time-frequency plot are extracted. Figure 2(a), (b) and (c) show the free decay displacement response at the top storey level after an earthquake, its time-frequency plot and the extracted window parallel to the frequency axis at the wavelet ridge, respectively. It is clear from Figure 2(b) and (c) that the peak value of the modulus of CWT coefficients corresponds to the frequency of 2.1Hz which is very close to the 1st mode frequency obtained from the modal analysis. The difference is less than 1%.

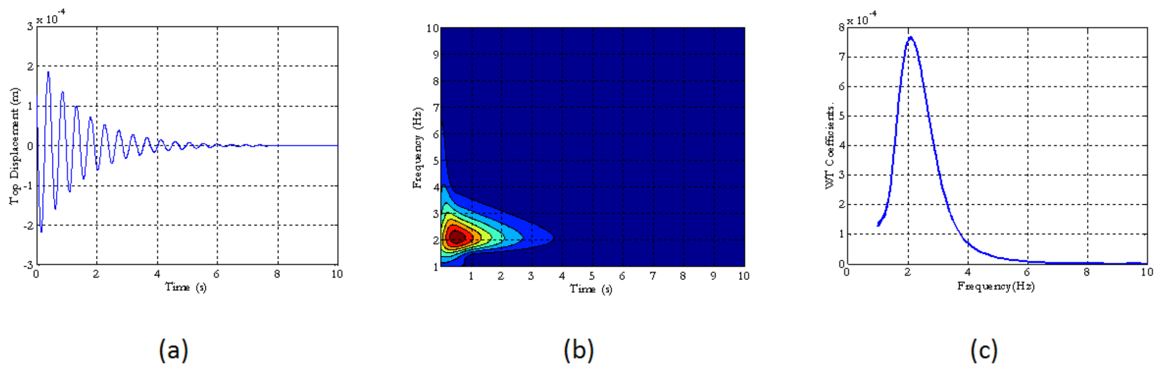

Figure 2. (a) Free decay response at top story, (b) its time-frequency resolution and (c) the extracted window parallel to frequency axis at the wavelet ridge

Furthermore, a window paralleled to the time axis at the wavelet ridge is extracted from the time-frequency plot to estimate the damping ratio of the corresponding mode of vibration. Figure 3(a) and (b) show the extracted envelop of the CWT coefficient modulus at the frequency of 2.1Hz and its semi-logarithmic plot, respectively. Furthermore, Figure 3(c) shows the comparison between the free decay response and the extracted envelop. They are in good agreement proving that WT coefficients are proportional to the instantaneous amplitude of the response. The estimated damping ratio corresponding to the 1st mode is 5.09% which is very close to the assumed damping ratio of 5%. The difference in the estimation is less than 2%. There is the maximum error of 1% in the estimation of first mode frequency while it is 2% in estimation of damping ratio of the first mode vibration.

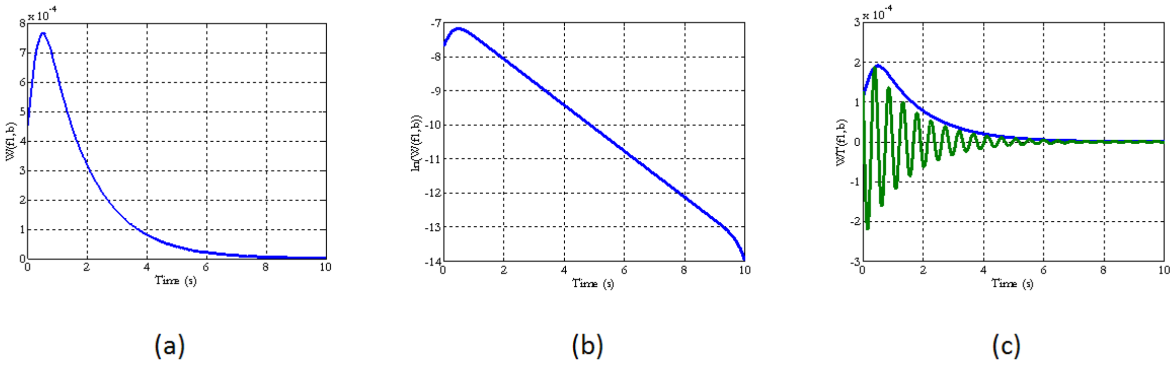

Figure 3. (a) the extracted envelope parallel to frequency axis at the wavelet ridge, (b) semi-logarithmic plot of the envelope and (c) comparison of free decay response and the wavelet envelop

4. ERROR IN DAMPING RATIO ESTIMATION USING TWO STEPS PROCEDURE SETUP

Figure 4 summarizes the two step procedure to estimate damping ratio in this study. Displacement response at the first storey level resulting in the input ground motion is obtained and subsequently, the RD signature is evaluated using the RD method as shown in the step 1(b). It should be noted that the level crossing triggering technique, which is commonly used in the literature, is adopted in this study. 214 number of segments are used in evaluation of the RD signature shown in Figure 4.

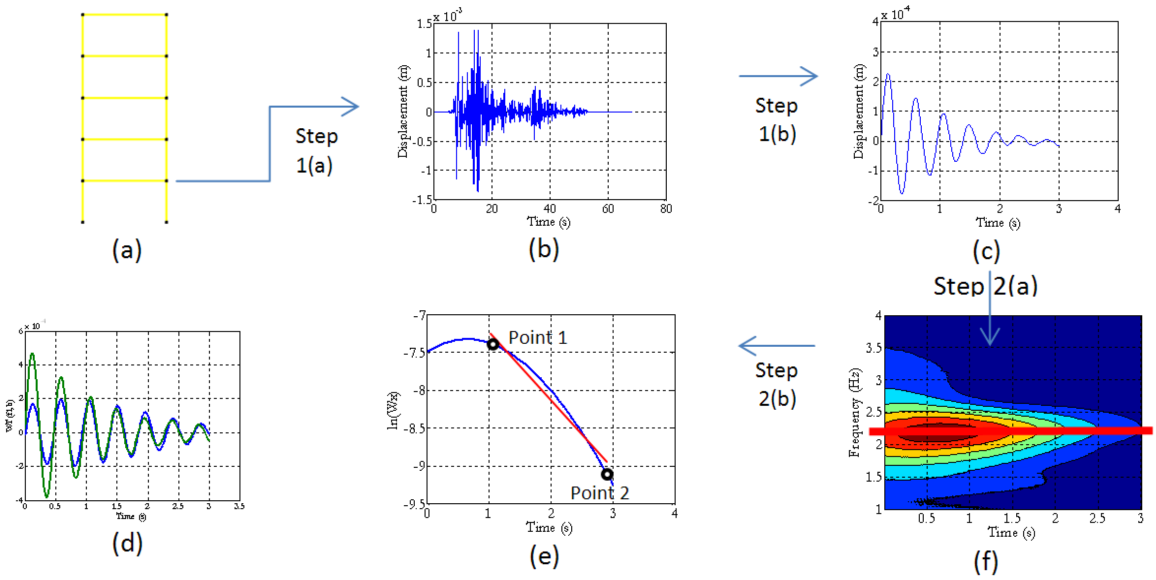

Figure 4. The two step procedure for estimation of damping ratio

The RD signature is then decomposed into time-frequency domain using CWT as shown in Step 2(a). In the step 2(b), a window parallel to the time axis is extracted at the wavelet ridge and then the semi-logarithmic plot of the CWT coefficient modulus is obtained. It is clear that the straight line as shown in Figure 4(b) is not observed. As a consequence of that a straight line is approximated using linear regression to the portion of the curve between the two points that are indicated in the black circles in

Figure 4. As shown in Figure 4(d), the amplitudes of the first two cycles are not matched well with the wavelet due the edge effect. Therefore, the first point represents the third peak where the amplitude of the response and the real part of the scaled WT coefficient are very close to each other and the second point represents the peak of the last cycle of the response. Finally, the estimated damping ratio is 5.93% which is 18.7% higher estimation than the assumed damping ratio of 5% for the first mode. Table 1 presents the earthquake no., duration of the ground motion, number of segments used to evaluate the RD signature, the duration of the RD signature, estimated damping ratio, estimated period, error in estimation of damping ratio and the error period of each earthquake. Figure 5 indicates that the maximum percentage of the error in the estimation of damping ratio is 26%.

Table 1. Details of the properties

Earthquake No.	Duration (s)	No. of segments	Duration of RD sig. (s)	Damping ratio (%)	Period (s)	Error in period estimation (%)	Error in damping estimation (%)
1	53	126	5,0	4,844	0,480	2,1	-3,1
2	53	122	3,0	5,233	0,457	-2,8	4,7
3	27	84	5,0	3,980	0,472	0,4	-20,4
4	27	104	3,0	4,944	0,485	3,2	-1,1
5	22	81	3,0	4,648	0,468	-0,4	-7,0
6	32	102	7,0	4,598	0,428	-4,6	-8,0
7	42	74	3,0	4,776	0,473	0,7	-4,5
8	34	144	3,0	4,469	4,950	5,3	-10,6
9	38	142	1,5	4,396	0,485	3,2	-12,1
10	26	82	3,5	4,436	0,502	6,7	-11,3
11	30	76	2,0	4,587	0,485	3,2	-8,3
12	42	100	5,0	4,197	0,490	4,3	-16,1
13	37	134	2,5	5,277	0,465	-1,1	5,5
14	38	78	2,0	4,529	0,463	-1,4	-9,4
15	49	154	3,0	4,117	0,472	0,4	-17,7
16	48	182	3,0	4,312	0,473	0,7	-13,8
17	56	150	3,0	5,520	0,472	0,4	10,4
18	88	292	2,0	4,514	0,473	0,7	-9,7
19	88	256	2,0	4,355	0,447	-5,0	-12,9
20	65	234	4,0	4,140	0,458	-2,5	-17,2
21	68	236	2,0	4,897	0,482	2,5	-2,1
22	68	214	3,0	4,990	0,458	-2,5	-0,2
23	73	228	5,0	5,598	0,475	1,1	12,0
24	73	214	2,0	4,805	0,465	-1,1	-3,9
25	23	76	2,0	4,650	0,498	6,0	-7,0
26	22	74	3,0	5,132	0,452	-3,9	2,6
27	103	306	3,0	4,433	0,483	2,8	-11,3
28	111	334	2,5	6,300	0,458	-2,5	26,0
29	111	356	2,0	5,003	0,447	-5,0	0,1
30	98	258	2,5	4,959	0,458	-2,5	-0,8
31	67	188	3,0	5,933	0,460	-2,1	18,7
32	67	186	3,0	4,336	0,475	1,1	-13,3
33	59	178	2,5	4,732	0,485	3,2	-5,4
34	102	292	3,0	4,433	0,478	1,8	-11,4
35	46	132	4,0	4,160	0,472	0,4	-16,8
36	33	84	3,5	5,460	0,485	3,2	9,2
37	43	116	4,0	4,854	0,458	-2,5	-2,9
38	47	136	2,0	4,807	0,505	6,4	-3,9
39	70	246	2,5	5,690	0,485	3,2	13,8
40	47	160	3,5	5,423	0,474	0,9	8,5
41	67	246	2,5	4,424	0,465	-1,1	-11,5
42	67	226	3,0	5,338	0,465	-1,1	6,8
43	40	100	4,0	4,639	0,463	-1,4	-7,2
44	39	120	5,0	4,232	0,482	2,5	-15,4
45	48	130	2,0	5,627	0,437	-5,0	12,5
46	67	200	2,5	4,747	0,458	-2,5	-5,1

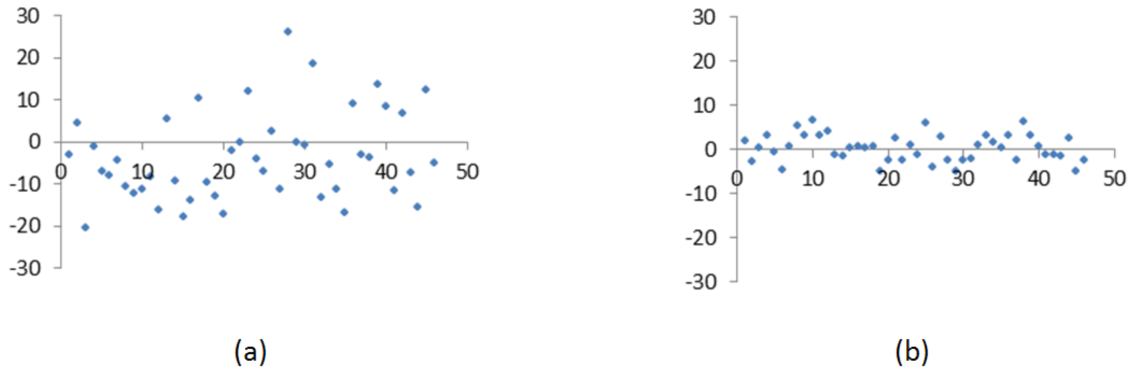

Figure 5. Variation of (a) damping ratio (b) period

4. ERROR IN PERIOD ESTIMATION

Figure 6 illustrates the single step procedure used to estimate the natural frequencies of the structure. The displacement response at the first story level shown in Figure 6(a) is decomposed into a time-frequency domain. Figure 6(b) shows the time-frequency plot. The periods are estimated using an extracted window parallel to the frequency axis at a wavelet ridge. Figure 6(c) indicated the wavelet ridge at which the window is extracted. Figure 5(b) shows the variation of error in the estimation of 1st mode frequency of the structure for different ground motions selected in this study. It indicates that the maximum error in the estimation of period is 6.7%. This highlights the capability of CWT method with complex Morlet wavelet to estimate the period adequately accurate even using a response which is non-stationary and lasts in relatively short duration.

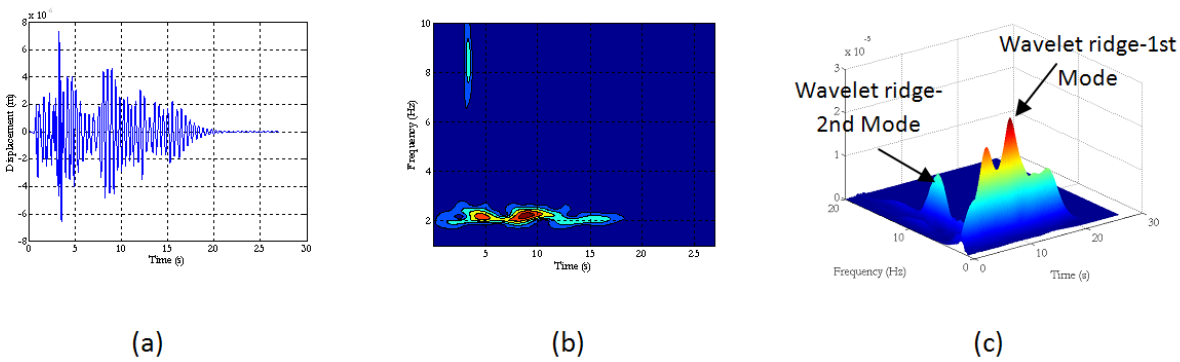

Figure 6. (a) The displacement response at the first story level, (b) its time-frequency plot and (c) wavelet ridge

Furthermore, Figure 6(b) illustrates that the frequency of the second mode of vibration can also be extracted. The extracted frequency is 8.3Hz which is very close to the second mode period of 8.2Hz obtained from the modal analysis.

Figure 7 shows the comparison of the first and the second mode shapes extracted using CWT and the modal analysis. They are in very good agreement.

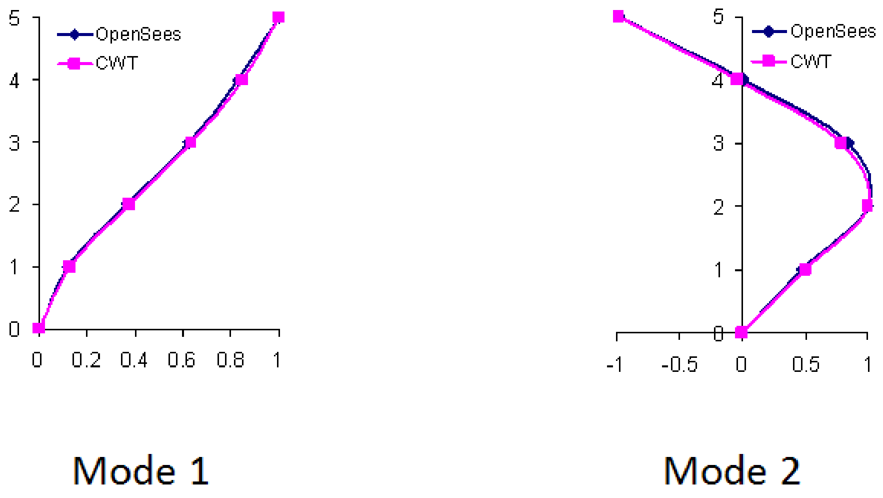

Figure 7. Comparison of the mode shapes obtained using numerical model and the CWT

5. CONCLUSIONS

This study mainly investigates the capability of the continuous wavelet transformation (CWT) method using Morlet wavelet in estimation of modal properties of low-rise buildings. For this purpose, a numerical is used. Based on the results of the application following conclusions can be drawn.
It can be concluded that the CWT method can estimate the modal properties accurately when the free decay response of a MDOF system is analyzed. Furthermore, when a response is non-stationary and lasts in relatively short duration, the CWT method can estimate the periods and mode shapes adequately accurate. However, a very significant scatter in estimation of damping ratio is observed with the maximum error of 26% when the two steps procedure is used. This could be due the fact that in damping estimation, the evaluation of RD signature for such a non-stationary and short duration record does not represent the free decay response of the structure.

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REFERENCES

Bedewi N. E., “The mathematical foundation of the auto and cross-random decrement techniques and the development of a system identification technique for the detection of structural deterioration”, Ph.D. Dissertation, Department of Mechanical Engineering, University of Maryland, College Park, U.S.A., 1986.
Brincker R., Zhang L., Andersen P., “Modal identification of output-only systems using frequency domain decomposition”, Smart Materials and Structures, Vol. 10, pp. 441-445, 2001.
Brincker R., Ventura C.E., Andersen P., “Damping estimation by frequency domain decomposition”, Proceedings of the 19th Int’l Modal Analysis Conference (IMAC), pp.698-703, 2001.
Cole H.A., “On-line Failure Detection and Damping Measurements of Aerospace Structures by Random Decrement Signature”, NASA CR-2205, 1973.
Hans S., Ibraim E., Pernot S., Boutin C., Lamarque C. H., “Damping identification in multi-degree-of-freedom systems via a wavelet-logarithmic decrement—part 2: study of a civil engineering building”, Journal of Sound and Vibration, Vol. 235, pp. 375–403, 2000.
Huang C. S. and Yeh C.H., “Some Properties of RandomDec Signatures”, Mechanical Systems and Signal

Processing, Vol. 13(3), pp. 491-507, 1999.
Ibrahim, S. R. "Random Decrement Technique for Modal Identification of Structures," Journal of Spacecraft and Rockets, Vol. 14, pp. 696-700 , 1977.
Juang J. N., Pappa R. S., "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction", AIAA Journal of Guidance, Control and Dynamics, Vol. 8, No. 4, pp. 620-627, 1985.
Kareem A. and Gurley K., "Damping in Structures: Its Evaluation and Treatment of Uncertainty", Journal of Wind Engineering and Industrial Aerodynamics, Vol. 59, pp. 131-157, 1996.
Le T. H., Tamura Y., "Modal Identification of Ambient Vibration Structures Using Frequency Domain Decomposition and Wavelet Transform", Proceedings of the 7th Asia-Pacific Conference on Wind Engineering, Taipei, Taiwan, November 8-12, 2009.
Lamarque C. H., Pernot S., and Cure S., "Damping Identification in Muti-Degree-of-Freedom Systems via a Wavelet-Logarithmic Decrement-Part 1: Theory", Journal of Sound and Vibration Vol. 235(3), pp. 361-374, 2000.
Staszewski W.J., "Identification of Damping in M.d.o.f. Systems using Time-scale Decomposition, Journal of Sound and Vibration, Vol. 203, pp. 283–305, 1997.
Ta M. N., Lardiès J., "Identification of Weak Nonlinearities on Damping and Stiffness by the Continuous Wavelet Transform", Journal of Sound and Vibration Vol. 293, pp. 16-37, 2006.
Vandiver J. K., Dunwoody A. B., Campbell R. B. and Cook M. F., "A mathematical basis for the random decrement vibration signature analysis technique", Journal of Mechanical Design, Vol. 104, pp. 307-313, 1982.
Yang J. C. S., Dagalakis N. G., Everstine J. C. and Wang Y. F., "Measurement of structural damping using the random decrement technique", Shock Vibration Bull., vol. 53 (4), pp. 63-71, 1983.