A Coupled 2D Finite-Elements - Finite-Difference Method for Seismic Wave Propagation: Case of a Semi-Circular Canyon

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Abstract

We present a coupled method to simulate elastic wave propagation when taking into account the topography of a free surface. This method combines a finite-elements method (FEM) and a 4^{th}-order velocity-stress staggered-grid finite-difference method (FDM). The aim of this combination of two numerical methods is to keep the advantages of both methods. The FEM is used to model the topography where the FDM cannot accurately model it. To couple the two methods, a finite-width interface (5 finite-difference grid spacing) is prepared, where a high-order interpolation scheme is used. The accuracy and efficiency of the method are tested by the simulation of wave propagation in a semi-circular canyon. Then, more complex models are studied.

Introduction

The 4^{th}-order velocity-stress staggered-grid finite-difference method (FDM) is a simple and efficient method to simulate seismic wave propagation in elastic media and was therefore widely implemented [1]. However, it cannot accurately model free surfaces and topography. The finite elements method (FEM) is a more accurate method to model the free surface and the topography because it can be used with irregular grids and elements of different sizes and geometries. However, it requires significantly more memory than the FDM. To keep the advantages of both FDM and FEM, a combination of both methods was proposed with 2^{nd}-order finite-difference [2] and with 4^{th}-order finite-difference [3]. We adopt Ma et al.'s method [3] to simulate wave propagation in a semi-circular canyon.

1. Method

1.1. Geometry

Model geometry of the validation model is shown in Figure 1. For the FD part, we use a classical 4^{th}-order velocity-stress staggered-grid FDM [1, 4]. At the boundaries, we used a Convolutional Perfectly Matched Layer (CPML, see [5]) to avoid artificial reflections (grey area). For the FE part, we prepare two domains:

- A layer of structured square meshes which size is half the size of the FD grid. This layer must overlap the FD grid and must be at least 5 finite-difference grid spacing wide.
- Ordinary unstructured meshes which can be either quadrangular or triangular elements.

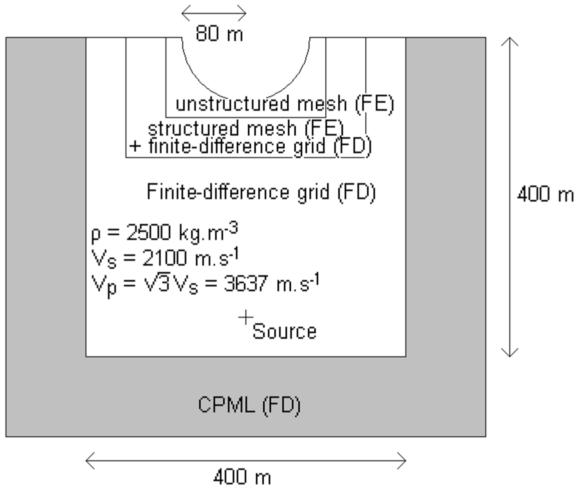

Figure 1. Model geometry tested in this study. The main part of the medium is modeled with finite-difference grid. The area around the canyon is modeled with an unstructured finite-element mesh. Between the two areas, an interface layer is modeled with a structured finite-elements mesh.

1.2. Procedure

We follow the same procedure as [3]. It is summarized as:

1. Calculate velocities at time n+1/2 from those at time level n-1/2 and stresses at time level n in the FD region.
2. Calculate displacements at time n+1 from those at time n and n-1 in the FE region. Using displacements at time level n and n+1, calculate the velocities at time n+1/2 in the interface area and assign them to the appropriate FD grid points.
3. Update the stresses at time level n+1 from those at time level n and velocities at time level n+1/2 in the FD and interface region.
4. As in Ma et al.'s paper [3], we assume the velocity field as a 3rd-order polynomial, which give us an interpolation scheme to get the velocities at FE mesh points on the boundary of the interface. Calculate the displacements at time level n+1 from displacements at time level n and velocities at time level n+1/2.

2. Result

We use a double-couple point source with the following release-rate function:

with : $m_0 = 10^{13}$ N.m.s^{-1}, t_s = 0.1 s and t_p = 0.05 s. The source is located in the middle of the medium at y = - 200 m. The FD grid spacing is 10 m. The time step is 0.0001 s. We show in Figure 2 snapshots of the horizontal and vertical velocity at time steps 0.16 s, 0.20 s, 0.24 s, 0.28 s and 0.32 s. The S-wave arrives at the bottom of the canyon at around 0.20 s. Then, we observe the up-going S-wave along the canyon shape and the reflected S-wave going back to the bottom. The strong inflected waves are due to the extremely sloped edges of the canyon (t = 0.28 s). This simulation shows the good efficiency of the FD-FE coupling. The method is then applied to more complex models.

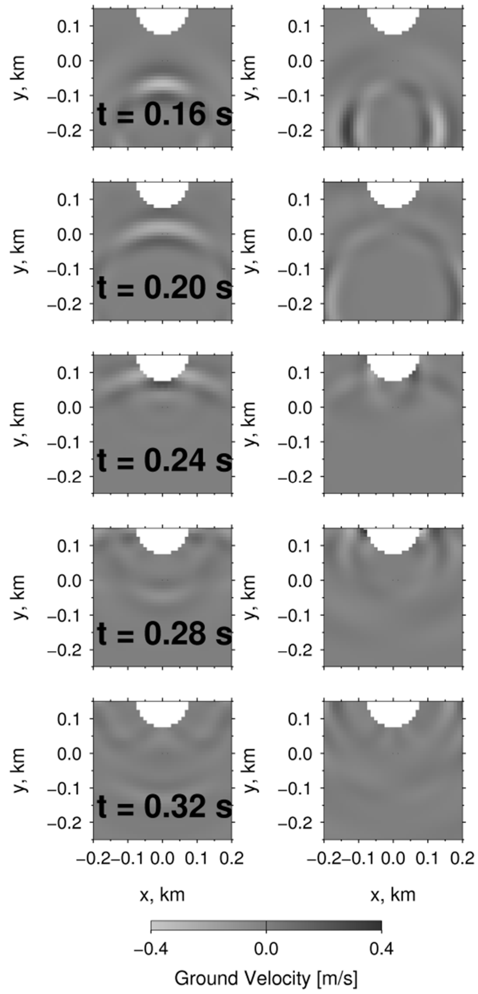

Figure 2. Snapshots of the horizontal (left) and vertical (right) velocity in the medium at time steps 0.16 s, 0.20 s, 0.24 s, 0.28 s and 0.32 s (top to bottom). The CPML area is not shown here.

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