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Spatial Extreme Value Analysis of Significant Wave Heights Along the French Coast

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Abstract

Extreme value analysis is of paramount importance in coastal engineering, for structure design as well as hazard mapping. The significant wave height (SWH) is the parameter generally used to characterize the intensity of sea states. Extreme value analysis on SWH requires historical buoy records of sufficient length and good quality. However, such observation datasets are often inexistent and numerical hindcasts of waves are used instead. One advantage of using such model outputs is that an extreme value analysis over a large spatial area is possible, enabling one to highlight spatial variations on extremes.

In this study, we aim at studying spatial variations of extreme values of SWH along the French coast for current climate. We use wave data from the BoBWA-10kH database (Charles et al., 2012) which is a numerical wave hindcast for the whole west coast, performed with the third generation wave model WIII (Tolman, 2009) and forced with ERA-40 reanalysis winds. It covers the period 1958-2002 and has a spatial resolution of 10km. Extreme value analysis is performed for about 40 points regularly distributed along the coast. The Peaks-Over-Threshold method is used and a Generalized Pareto Distribution is fitted to the data. Spatial variations along the coast for several return values of SWH (10-year, 50-year, 100-year) are presented and discussed.

1. INTRODUCTION

In coastal engineering, extreme value analysis is widely used for various applications, from flooding hazard mapping to the design of marine works. It is a way to project oneself in the future to get a sense of “what are the odds that this event happen?” or “which event has an occurrence probability p?”. The intensity of sea states is generally characterized by the significant wave height (SWH), which is traditionally defined as the mean wave height (trough to crest) of the highest third of the waves. Good quality long time series of SWH are required to perform a sound statistical analysis of extremes. However, the available historical buoy records along the French coast are scarce and often discontinuous, with numerous gaps occurring during extreme events like storms. To overcome this issue, numerical hindcasts of waves can be used instead. Such models are usually calibrated and validated against buoy or satellite data to provide an accurate representation of reality. Nevertheless, it is worth noting that as soon as one uses model outputs to calculate extreme values, uncertainties linked to errors and approximations (inherent to wind data and wave model simplifications) are introduced. The main advantages of using such model outputs remain (1) the length of the time series that enables one to calculate higher extreme values and to reduce confidence intervals and (2) the possibility to highlight relative spatial variations of extreme values thanks to the large spatial area covered by the data.

Numerous statistical methods exist to determine extreme wave height. Among the most commonly used, the Peaks-Over-Threshold (POT) approach has the advantage of using all the available information on extremes behavior of the time series as soon as a suitable threshold is determined. A natural candidate for the probability distribution is then the Generalized Pareto Distribution (GPD) which is the most general form of the distribution for POT samples. This method (POT-GPD) is widely used and recommended to calculate extreme values of SWH (e.g. Hawkes et al, 2008; Mazas & Hamm, 2011; Li et al, 2012). Nevertheless the choice of the threshold and the determination of the best law are always delicate issues, depending on the tail of the distribution. These two points must be discussed, especially when one wants to work at regional scale with a homogeneous method in order to analyze spatial variations.
Presently, ANEMOC (Numerical Atlas of Oceanic and Coastal Sea States) is the only available database of wave extreme values covering the French Atlantic and Mediterranean coasts with a good point density (Benoit et al., 2006). It was realized by EDF/LNHE and the CETMEF from a wave hindcast based on the third generation model TOMAWAC (Benoit et al., 1996) and the ERA-40 winds (Uppala et al., 2005). It covers the period 1979-2002. A recent study of wave hindcasts intercomparison (Lecacheux & Paris, 2013) pointed out that this dataset presents a positive bias for values above the 90th percentile compared to observations. Yet, extreme value analyses are very sensible to events constituting the tail of the distribution and we can expect the ANEMOC extreme values to be overestimated.

In this study, realized for the French Ministry of Environment, we performed a spatial extreme value analysis of SWH along the French Atlantic coast using an alternative wave hindcast, namely the Bay of Biscay Wave Atlas (BoBWA-10kH) database (Charles et al., 2012), and the POT-GPD method. This paper presents the preliminary results and is organized as follows: section 2 introduces the BoBWA-10kH database and the statistical analysis method; in section 3, we present the preliminary results and a comparison with the ANEMOC database; finally, section 4 is dedicated to the discussion and the conclusion. The final results of the project should be available at the end of the year.

2. DATA AND METHOD

2.1. Description and validation of BoBWA-10kH

BoBWA-10kH (Charles et al., 2012) is a wave hindcast covering the period 1958-2001. It was realized with a two-way nested Wavewatch 3 (Tolman, 2009) modeling framework covering the North Atlantic (spatial resolution of 0.5°) and the French Atlantic and English Channel coasts (spatial resolution of 0.1°), and using the parameterization of Ardhuin et al. (2009). The model was forced by ERA-40 reanalysis winds (Uppala et al., 2005) given every 6 hours at a height of 10m on a 1.125°X1.125° grid. In the simulations, the water level is supposed to be constant (mean level) and the currents are not taken into account. A calibration was carried out at the Biscay buoy on the period 1998-2002 by varying the wind input height. The results were stored hourly at the buoy locations along the coast and every six hours for each point of the grid.

The validation performed by Charles et al. (2012) on 9 buoys showed a good agreement with observations for the Atlantic coast but a poorer quality of the model in the English Channel. This fact was attributed to the coarse resolution of the model that prevents the proper modeling of waves coming from the North Sea and the fact that interactions with the strong tidal currents in this area was not taken into account. Lecacheux et al. (2013) showed that, in the area of the Bay of Biscay, BoBWA-10kH had the lowest statistical errors compared to the two other available regional hindcasts (ANEMOC and Bertin & Dodet, 2010). The highest values of wave heights (above the 90th percentile) seemed also to be better reproduced.

For this study, we investigated the capacity of BoBWA-10kH to reproduce storm events (peak, length, etc.). We compared the model outputs with observations on common periods at one offshore buoy (Biscay) and two coastal buoys (Biscarrosse and Minquiers) presented on Figure 1. For each buoy, the storm events correspond to the periods for which SWH exceeds 2/3 of the maximum value reached during the entire record. With this technic, we detected 9 events at the Biscay buoy (from 1998 to 2002), 13 events at the Biscarrosse buoy (from 1998 to 2002) and 7 events at the Minquiers buoy (from 1992 to 1994 and from 1997 to 2002). The results show a good correlation between simulations and observations (R² = 0.87) and we do not notice any systematic bias (cf. Figure 1).

Concerning the peaks of the storms (which are of paramount importance in extreme value analysis), we noticed relative errors lower than 7% at Biscay and Minquiers but up to 17% at Biscarrosse. For this last buoy, the higher statistical errors can be attributed to its location close to the coast (< 5km) and the insufficient resolution of the model in this area.
Forty three points have been selected in the BoBWA-10kH dataset to perform the statistical analysis (Figure 2): 31 grid points (6 hourly) and 12 buoys locations (hourly). They are evenly spaced of about 40-50 km along the coast and they are located about 50 km from the shore (except some coastal buoys). Nevertheless, the resolution of the model and the variations of the bathymetry along the coast did not allow selecting points at the same depth everywhere. Near the Aquitaine coast, the depth is about 50m; along the Brittany peninsula, it is around 100m; and in the English Channel it is around 30m.

2.2. Method to derive the GPD

To derive extreme values from significant wave heights (SWH) time series, we first need to identify independent and identically distributed (i.i.d.) events. A simple directional analysis is used to determine whether the highest SWH are associated with several discontinuous directional sectors or not. In the first case, that means the recorded storms of a given directional sector are generated by a
different kind of depressions than the ones of another sector. Therefore each storms group should be
 treated separately from the others as they may not be identically distributed. In practice, only a few
 points in the North of the English Channel have two key directional sectors. The remaining points only
 have one. The independence of events is achieved using a Peaks-Over-Threshold (POT) approach
 combined with a temporal criterion: a minimum period of 72 hours between each peak is chosen to
 consider them as independent events. The POT threshold \( u_t \) is determined roughly so as to select
 both weak and strong storms (which should represent a few hundreds of peaks). A statistical
 goodness-of-fit test (\( \chi^2 \)) enables us to make sure the annual occurrence of selected peaks follows a
 Poisson distribution at the risk level of 0.1 (when the test failed, a higher value for \( u_t \) was chosen). A
 higher threshold \( u_2 \) above which storms have a statistically extreme behavior is then chosen more
 thoroughly using several tests and plots. We follow here the double threshold method of Mazas &
 Hamm (2011).

The choice has been made to fit only the Generalized Pareto Distribution (GPD) to represent the
 distribution of extreme wave heights along the French coast:

\[
P(SWH - u_2 > y | SWH > u_2) = \begin{cases} 
(1 + \frac{\xi y}{\sigma})^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\
\exp \left( -\frac{y}{\sigma} \right) & \text{if } \xi = 0 
\end{cases}
\]

(1)

Where \( s_* = \max(s,0) \), \( \xi \) is the shape parameter and \( \sigma \) is the scale parameter.

It is indeed recommended to use it (Hawkes et al, 2008; Li et al, 2012) and since the aim of the study
 is a spatial analysis of extremes, it is important to be consistent and use a single statistical law for all
 the study sites. To determine \( u_2 \) we take advantage of the asymptotical properties of the GPD: if the
 sample follows a GPD then the mean excesses of SWH above \( u_2 \) vary linearly with \( u_2 \) (mean residual
 life plot) and the modified scale parameter \( \sigma^* = \sigma - \xi u_2 \) and the shape parameter \( \xi \) remain constant
 when \( u_2 \) increases. In practice, we fit the GPD for thresholds comprised between \( u_t \) and a threshold
 corresponding to 1 event per year and search for the lowest threshold of the highest domain of
 linearity resp. stability (Mazas & Hamm, 2011). However, despite these theoretical properties,
 determining the high threshold \( u_2 \) is not always straightforward. Sometimes several values might
 apply. In addition to these two plots, we thus perform two statistical tests namely the Kolmogorov-
 Smirnov and \( \chi^2 \) tests for the whole range of thresholds in order to validate or invalidate the values
 previously determined. We focus on maximizing the \( p \)-value of the two tests and we reject the
 thresholds when the tests have failed (with a risk level of 0.1). Last, we draw a sensibility graph
 representing the variations of 100-year SWH (SWH\(_{100}\)) with respect to \( u_2 \) (Mazas & Hamm, 2011).
 SWH\(_{100}\) shall remain roughly constant for thresholds above \( u_2 \). During the threshold selection process,
 we also try to follow a recommendation of Mazas & Hamm (2011) which is to prefer a value of \( u_2 \)
 corresponding to about 2 events per year when the number of years is large (over 40 years), which is
 the case in our study. All those information enable us to determine an adequate threshold most of the
 time. When it still remains difficult to choose between thresholds, we also use quantile-quantile plots
to make a final visual decision.

The estimation of the GPD parameters is also a crucial part of the analysis. Several methods exist, the
 most commonly used being the method of moments, the probability weighted moments and the
 maximum likelihood (Mackay et al, 2011). When it comes to choosing an estimator, three
 characteristics need to be considered: bias (is the expected value of the estimator equal to the
 parameter?), efficiency (is the variance (or RMSE) of the estimator as small as possible?) and
 consistency (as the number of observations \( n \) increases, does the estimator value approach the true
 parameter value?). To stay consistent all along the French coast and to be able to do comparisons
 between study sites, we decided to use only one method regardless of the relative performance of the
 different estimators. We chose the method of moments because the corresponding estimator is the
 only one having a small positive bias for \( n \) around 100 and \( \xi < 0 \), which is typically the case for our
 study, while keeping the RMSE reasonably low (Mackay et al, 2011). A small positive bias may
 compensate the slight underestimation of the GPD on grid points due to the 6-hour time step of the
 time series. Indeed, with a time discretization of 6 hours, the highest SWH in a given storm might be
 missing which can lead to an underestimation of extremes. A sensitivity test on the time step (1 hour
vs 6 hours) was therefore conducted on several buoys showing a difference in the results up to 3% on SWH$_{100}$.

Finally, confidence intervals are obtained from the classical Delta method (Coles, 2001).

All the analyses were done under MATLAB environment with the WAFO toolbox (Brodtkorb et al, 2000).

3. RESULTS

3.1. Example analysis of a study point

In this section, we go through the steps of the method to derive the GPD of SWH for the study point ‘Brittany_09’ located -5.4E, 48.5N (Figure 1). Figure 3 shows the waves’ characteristics of the dataset. Peak directions (Dp) are represented in nautical convention (incoming direction and North = 0°). SWH values can be seen on the radial axis. The envelope of the time series data points is represented by the dashed line. The occurrence frequency of (SWH, Dp) pairs is represented by the colorbar (min = 0.02‰) with a discretization of Dp every 5° and SWH every 5 cm. It can be seen that the highest waves come from the same directional sector (=230°-300°) so no directional analysis is required.

![Figure 3. Polar representation of waves’ characteristics for the point ‘Brittany_09’.](image)

The POT threshold $u_1$ is fixed at 8m, after checking the resulting population of storms follows a Poisson distribution, which corresponds to 196 values. Then we have to determine the best high threshold $u_2$. As described in section 2.2, we adjust a GPD to the data over a wide range of thresholds (from $u_1$ to a threshold corresponding to 1 event per year) and look at the stability of the shape parameter $\xi$ and of the modified scale parameter $\sigma^*$ (Figure 4a) and at the linearity of the mean residual life plot (Figure 4b). On Figure 4a we also display for each threshold, the corresponding number of events per year on the secondary axis.
Figure 4. (a) Stability of modified scale and shape parameters for the GPD. The vertical blue bars represent the 95% confidence intervals; (b) Linearity of the mean residual life plot. The dashed lines represent the bounds of the 95% confidence interval.

From both graphs, a value of $u_2$ around 9.25m seems adequate. This value corresponds to a number of events per year of 2 or so (in accordance with the recommendation of Mazas & Hamm, 2011). One can notice a significant variation in both graphs for a value of $u_2$ above 10m. However, the resulting number of events per year would be too small and so would be the corresponding number of remaining points to adjust the GPD. To validate our value of 9.25m, we plot the variation of the p-value obtained from the $\chi^2$ and Kolmogorov-Smirnov (KS) tests for the range of thresholds (Figure 5a). Both p-values are largely above 0.1. As a final check, we plot the variation of $\text{SWH}_{100}$ against $u_2$ (Figure 5b). $\text{SWH}_{100}$ can reasonably be considered as stable above 9.25m.

Figure 5. (a) Variation of the p-value of the two statistical tests $\chi^2$ and Kolmogorov-Smirnov with the threshold $u_2$; (b) Stability of $\text{SWH}_{100}$ with the threshold $u_2$.

The GPD is then adjusted to the 92 data points above $u_2$. Figure 6 shows the result with the Hazen plotting position for the data points.

Figure 6. Generalized Pareto Distribution for the point ‘Brittany_09’.
3.2. Spatial analysis of extremes of SWH

Figure 7 shows the results of the statistical analysis along the French Atlantic coast (except for 3 offshore buoy locations): spatial variations of $SWH_{10}$, $SWH_{50}$, and $SWH_{100}$ are thus highlighted. We can notice that the three quantities vary the same way: the lowest values are found in the English Channel, the highest ones around the Brittany coast and values in the middle are found along the Aquitaine coast and between Brittany and Normandy. This is partly explained by the bathymetry and the varying depth at each point of the study, as described in section 2.1. Figure 7 also displays the difference between $SWH_{100}$ and the maximum value of SWH simulated along the coast. These two quantities vary similarly along the coast with a range of differences from about 0.6m (in deep and relatively exposed areas such as the West of Brittany) to about 0m (in shallow and not very exposed areas such as the North of the English Channel or the Normandy coast to the East of Cotentin, but also in deep and more exposed area such as the South of Brittany). A difference close to 0m suggests that historical events have generated waves with SWH close to the 100-year value.

![Figure 7. Results of the statistical analysis for 40 points along the coast: (1) Values of SWH for return periods of 10 years (up left), 50 years (up right), and 100 years (down left) (2) Differences between $SWH_{100}$ and $SWH_{max}$.

3.3. Comparison with ANEMOC database

The spatial extreme value analysis performed with BoBWA-10kH dataset is then compared with the ANEMOC product. As mentioned in the introduction, it is likely that results of extreme value of SWH...
obtained with the ANEMOC database will be higher than those obtained with BoBWA-10kH. Indeed, Lecacheux & Paris (2013) showed that ANEMOC dataset presents a positive bias for values above the 90th percentile and that BoBWA-10kH compared better with observations for this range of values.

The extreme value analysis in ANEMOC is similar to the one presented in this paper. Storms were selected with a POT approach and two distributions were adjusted to the data: a GPD (with the maximum likelihood estimators) and the exponential distribution. To be able to compare results, we focus only on the GPD. However, since the meshes that were used in the models are different, both in nature and resolution, it was not possible to perform a comparison of perfectly co-localized points. In addition, the extreme analysis in ANEMOC was performed only for a selection of points. Nevertheless, we managed to do the comparison for 10 points along the coast (cf. green crosses on Figure 1 that represent ANEMOC points). Results are presented in Figure 8.

![Figure 8](image_url)

**Figure 8.** Comparison of extreme values obtained with ANEMOC and BoBWA-10kH. The indices of comparison points correspond to the green crosses on Figure 1. Vertical blue bars represent 95% confidence intervals on SWH\textsubscript{100} obtained with BoBWA-10kH.

We first notice the same regional tendencies between BoBWA-10kH and ANEMOC since the values vary similarly. Second, and as foreseen, values of SWH\textsubscript{100} of ANEMOC are higher than those obtained with BoBWA-10kH. The difference is up to 3-4m along the Aquitaine and Brittany coasts (points 1 to 6, between 50m and 100m depth) and around 2m in the English Channel (points 7 to 10, 30m depth). Even if the comparison is partial because of the above described constraints (no co-localization, influence of the depth on the results) that limit the number of comparison points, it still highlights a significant difference of about 2m in average all along the French Atlantic coast between the values of SWH\textsubscript{100} obtained with the two hindcasts. As it can be seen in Figure 8, ANEMOC values of SWH\textsubscript{100} are much higher than the upper bound of the calculated confidence intervals (except for points 4 and 5). Therefore, the difference cannot be explained with the help of probabilities. In addition, we also see that SWH\textsubscript{max} follows almost the same tendency than SWH\textsubscript{100} whatever the database. Thus, it appears that the initial raw data (model outputs) is of paramount importance for extreme value analysis and probably more important than practical questions related to statistics and probability theory (which distribution?, Which threshold?, etc.).

4. **DISCUSSION AND CONCLUSION**

The objective of this study was to perform a spatial extreme value analysis of SWH along the French Atlantic coast, taking advantage of a recent numerical wave hindcast BoBWA-10kH. That would offer an alternative to ANEMOC which is currently the only available database of wave extreme values covering the French Atlantic and Mediterranean coasts with a good point density (Benoit et al, 2006).

By comparing extreme values obtained with both datasets along the coast, we found the same general spatial pattern but BoBWA-10kH values were, in average, 2m below ANEMOC values. This result
underlines the crucial role of models and their calibration and validation to accurately represent storm peaks, in order to have good quality data and to perform a sound extreme value analysis.

Concerning the method to derive the GPD, we decided to use exclusively the method of moments to estimate the GPD parameters because it allows the comparison between study sites and it may compensate the slight underestimation of extremes due to the 6-hour time step for grid points (see section 2.2). If the goal was not to produce a regional map of extreme values but to derive extremes on a specific site, another method (such as maximum likelihood or probability weighted moments) might be more appropriate and we should perform a proper comparison of the results to choose the best method.

It is worth noting that the extreme value analysis is based on 44 years, from 1958 to 2001. Thus, several recent impacting storm events, such as Klaus in 2009 or Xynthia in 2010, are not taken into account in the analysis. For example, the Cap Ferret buoy (see Figure 1) recorded a value of SWH as high as 11.3m during Klaus in January 2009 (CETMEF, 2012). This value is about 90cm above the calculated SWH_{100} in our study. It is still below the upper bound of the 95% confidence interval (11.8m), but there is no doubt that if the time series had included 10 more years, the resulting extreme values would have been different. This observation raises another issue: since long numerical hindcasts such as BoBWA-10kH or ANEMOC are not regularly updated and completed with more data, how can we include all the available information on SWH in the extreme value analysis? How to combine time series with specific information (qualitative or quantitative information on historical events)? In the related field of hydrology, probabilistic methods have been developed over the last 30 years for the consideration of historical floods and revealed the real added value that represents the historical information, incomplete as it may be (Gaume et al, 2010 ; Payrastre et al, 2012 ; N’Guyen et al, 2013). The application of these methods in the field of coastal risks and more particularly in the study of extreme wave heights still falls within the area of research and should be investigated in future works to improve extreme value analyses.

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